IDEAL CORE OF REAL SEQUENCES AND LINEAR TRANSFORMATIONS

PAOLO LEONETTI

Given an ideal $\mathcal{I}$ on the natural numbers ω and a bounded real sequence $\boldsymbol{x}$, we denote by

$$\mathrm{core}_{\boldsymbol{x}}(\mathcal{I})$$

the smallest interval $[a,b]$ such that $\{n \in \omega : x_n \notin [a-\varepsilon, b+\varepsilon]\} \in \mathcal{I}$ for all $\varepsilon > 0$. Note that $\mathrm{core}_{\boldsymbol{x}}(\mathcal{I})$ corresponds to the interval $[\,\liminf \boldsymbol{x}, \limsup \boldsymbol{x}\,]$ if $\mathcal{I}$ is the ideal Fin of finite subsets of ω; in addition, $\mathrm{core}_{\boldsymbol{x}}(\mathcal{I}) = \{\eta\}$ if and only if $\mathcal{I}\text{-}\lim \boldsymbol{x} = \eta$.

Thus, given an infinite matrix $A = (a_{n,k} : n, k \in \omega)$ and ideals $\mathcal{I}, \mathcal{J}$ on ω, we provide several relationships between the $\mathcal{I}$-core of $\boldsymbol{x}$, the $\mathcal{J}$-core of the linear transformation $A\boldsymbol{x}$, and certain regularity properties of A. Incidentally, we show that the unit ball of $\ell_\infty(\mathbf{R}^d)$ coincides with the closed convex hull of its extreme points.

Università degli Studi dell'Insubria, Varese (Italy) and Università Bocconi, Milano (Italy)

Email address: leonetti.paolo@gmail.com