

On the Geographic Spread of Chikungunya between Brazil and Florida: A Multi-patch Model with Time-delay

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Brazil



UNICAMP

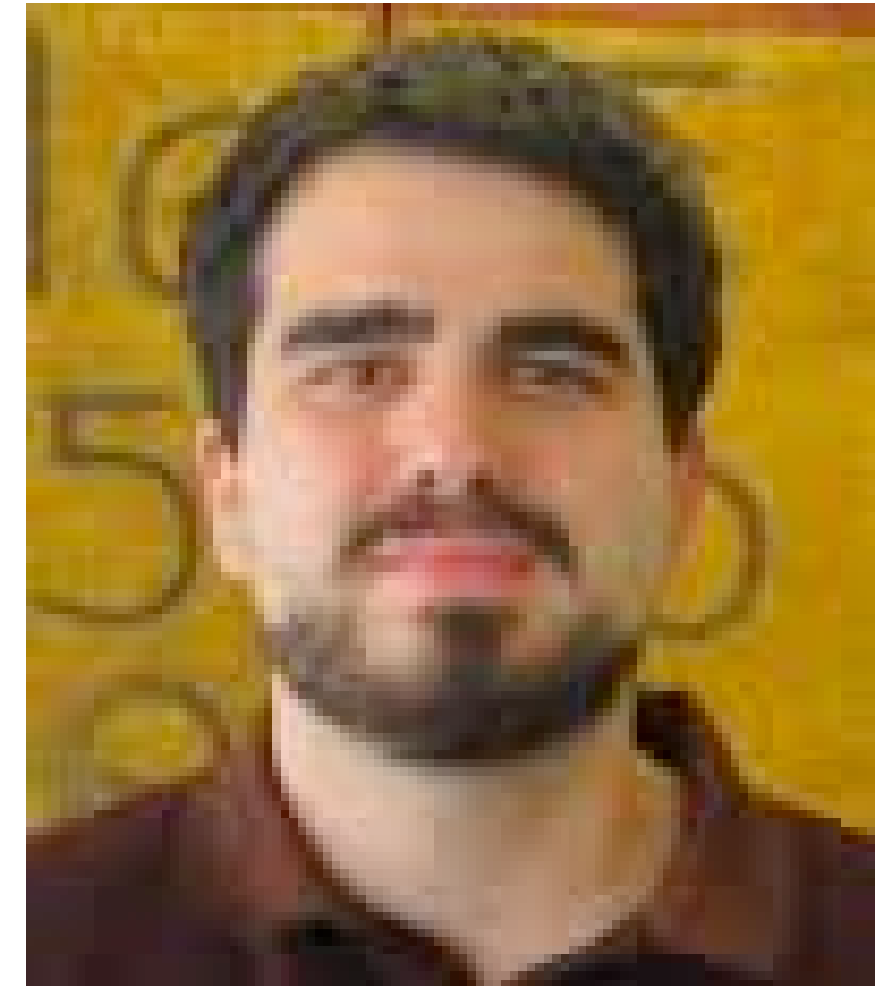
Co-authors



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University of Miami



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University of Miami

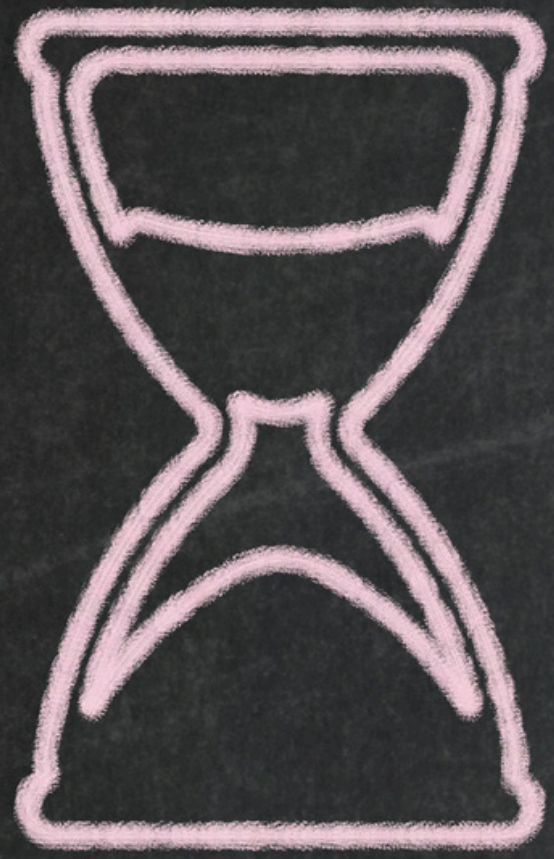


Antonio Gondim
University of Miami

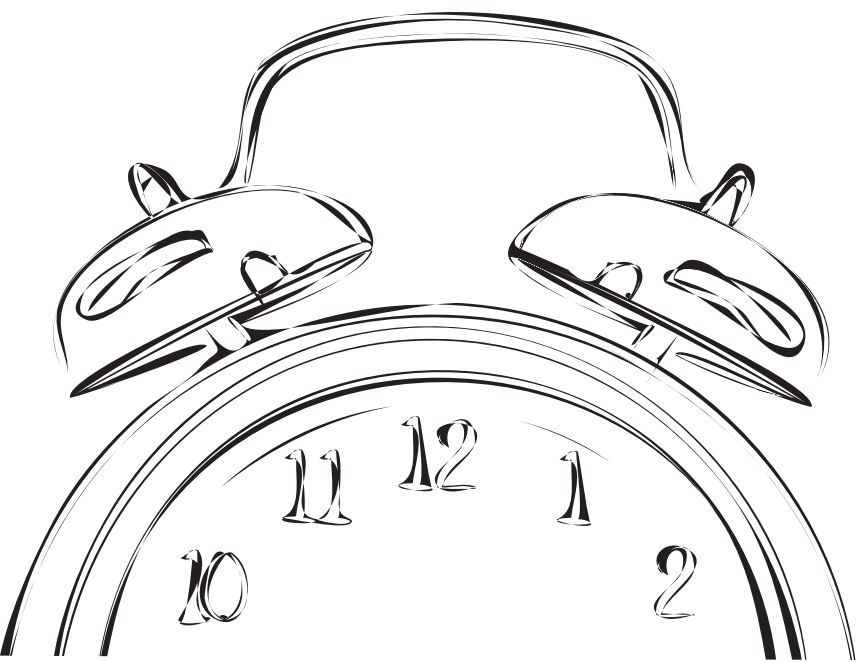
OUTLINE



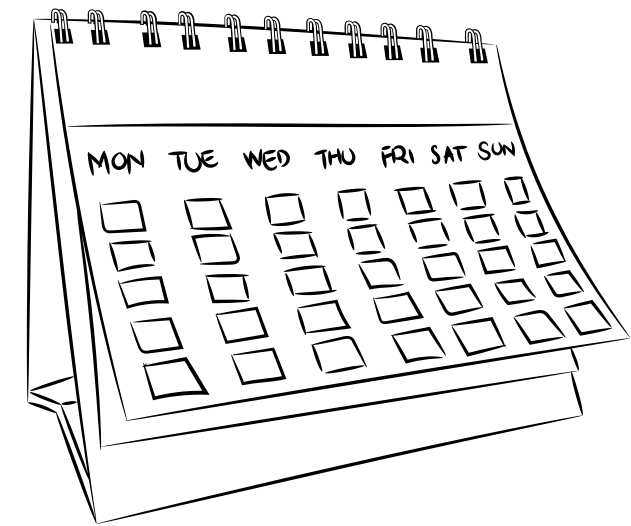
- **Timeline**
- **Motivation and applications**
- ▶ ■ **Problem**
- **Open Problems**



timeline



Time line



**Siméon Denis
Poisson**



1806

**Charles
Picard**



1906

Perez, Malta & Coutinho



1978

1771



Marquis of Condorcet

1888

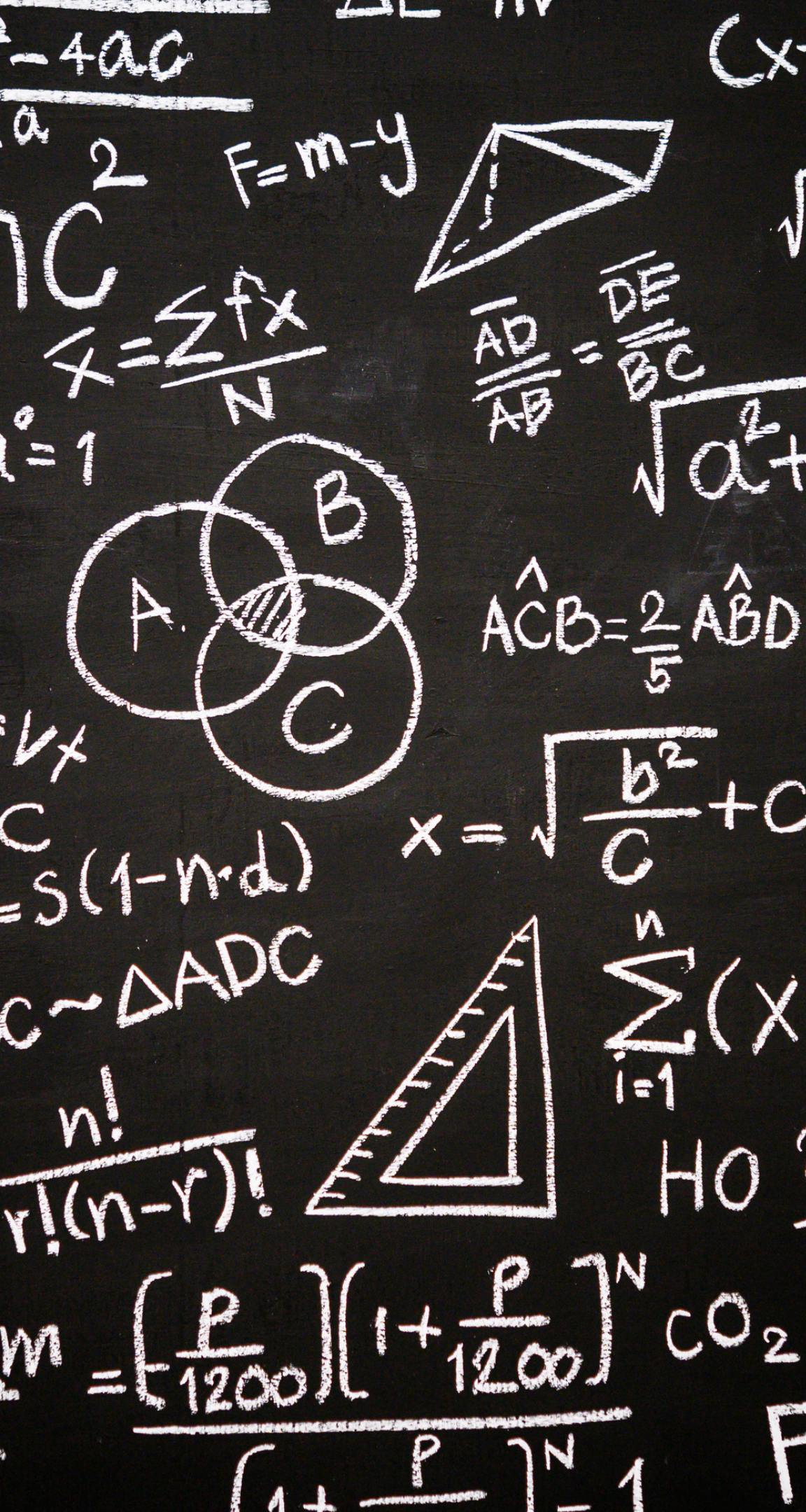


Vito Volterra

1948



George Hutchinson



Motivation

Delay equations

Population Model

Logistic Equation

$$\dot{x}(t) = bx(t) \left(1 - \frac{x(t)}{K} \right)$$



Pierre Franois Verhulst
(1838)

Population Model

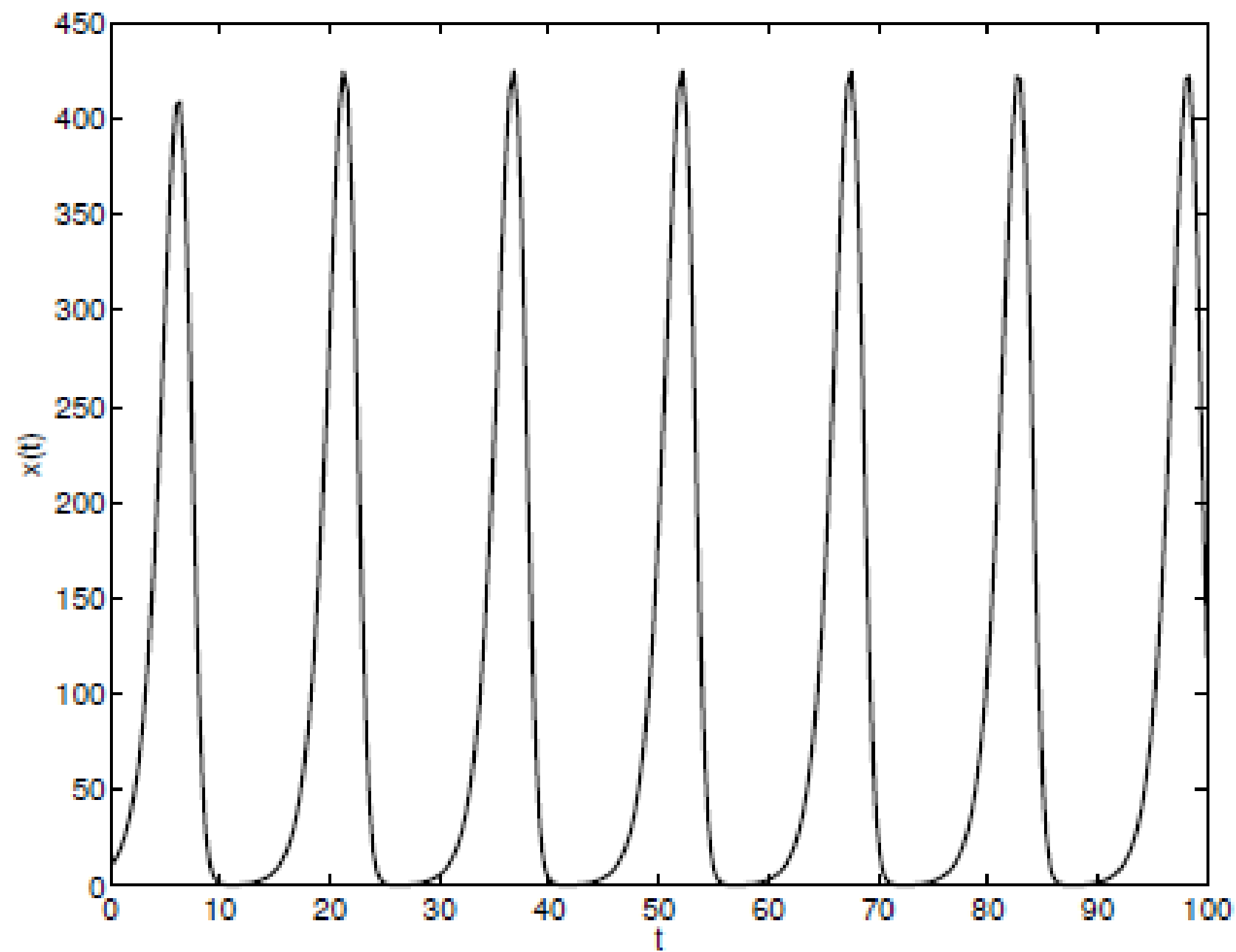
Delay equation

$$\dot{x}(t) = bx(t) \left(1 - \frac{x(t-\tau)}{K} \right)$$



George Evelyn Hutchinson
(1948)

Graphics

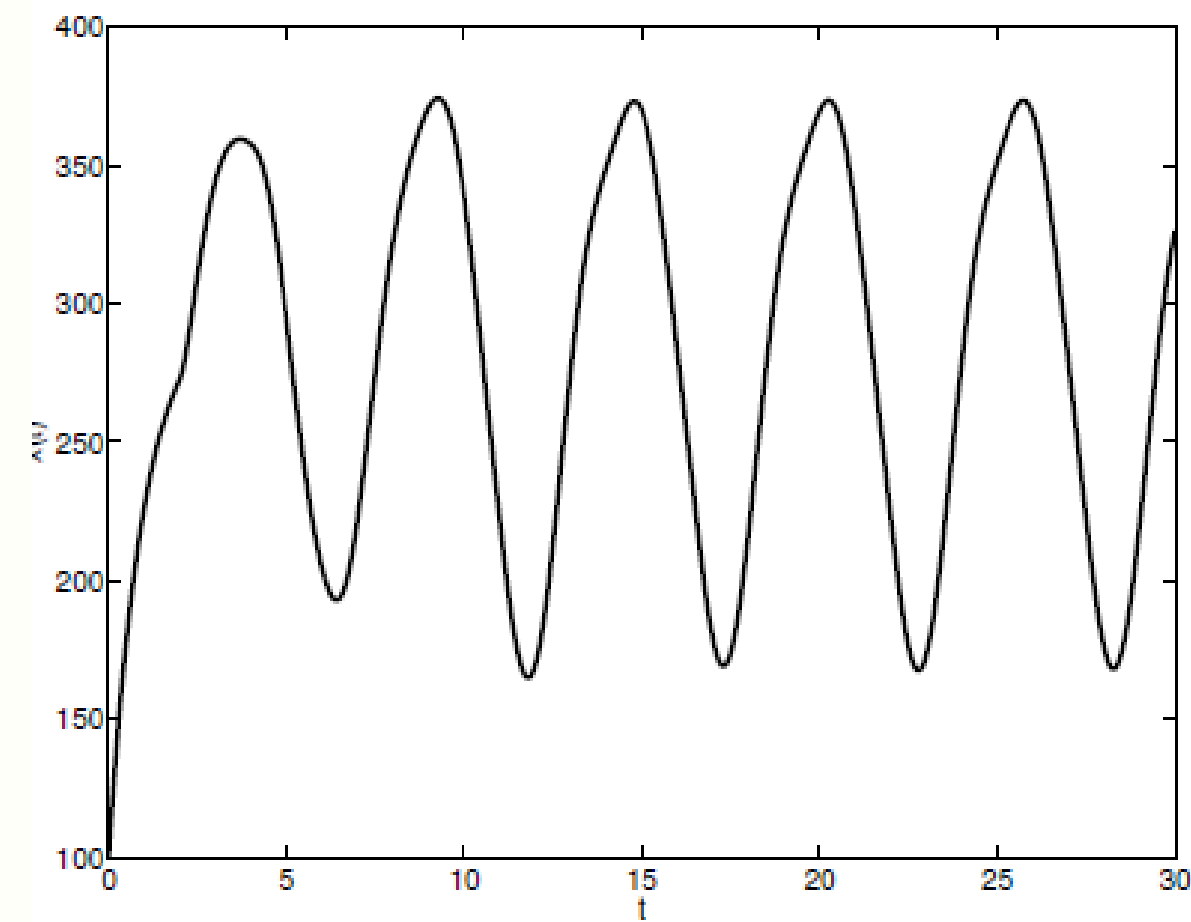
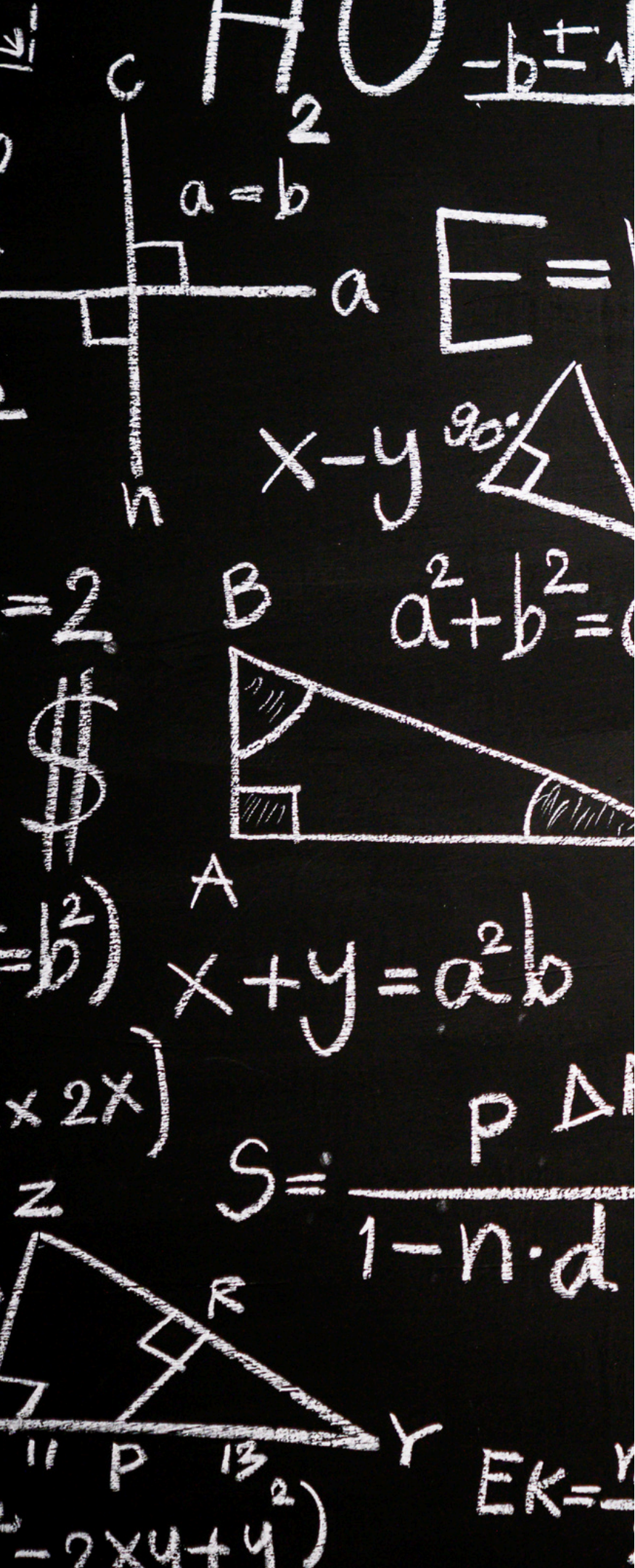


*Figura retirada de tese de doutorado de Maria Barbarossa

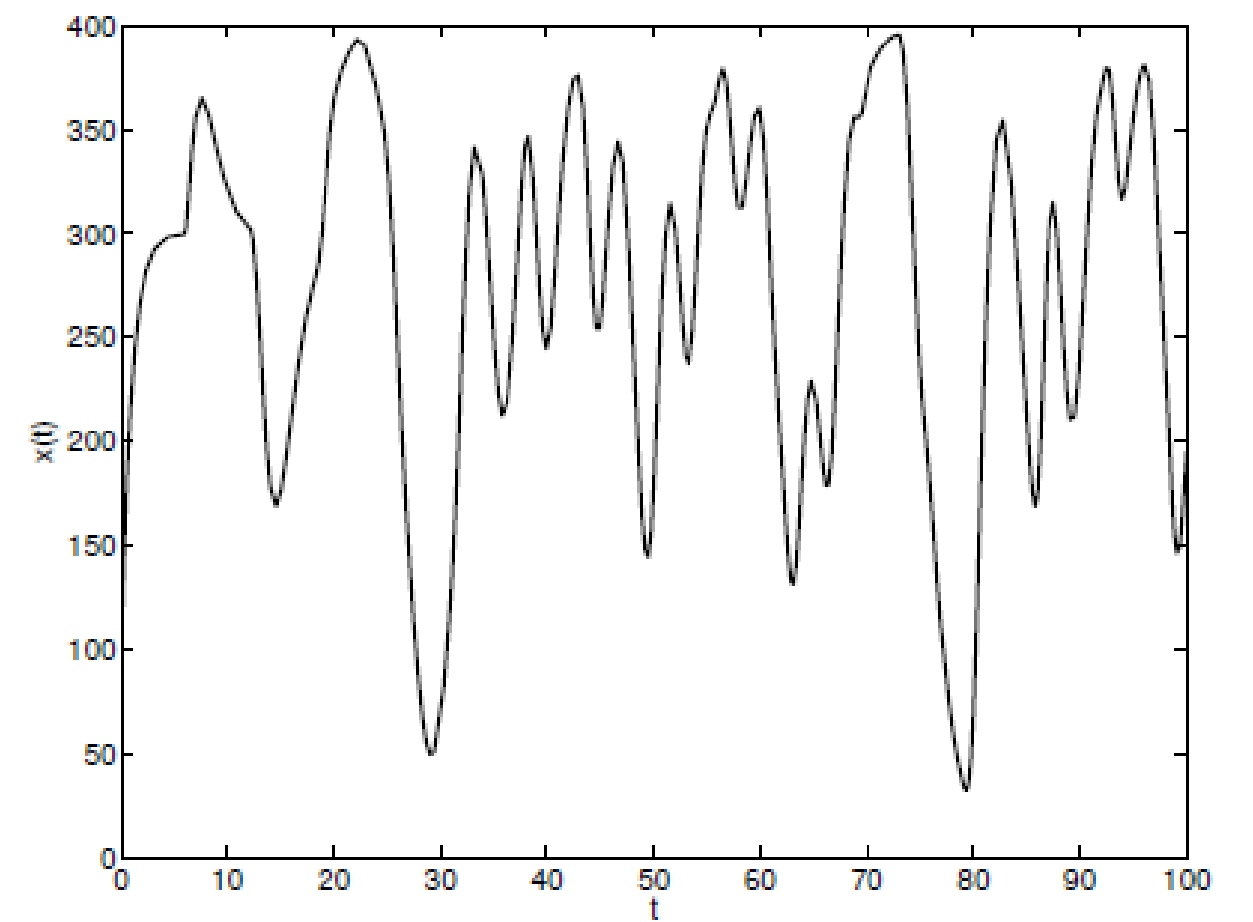
Blowfly Equation



$$\dot{x}(t) = b(x(t - \tau))x(t - \tau) - \mu(x(t))x(t)$$



(a) $\tau = 2$.



(b) $\tau = 6$.

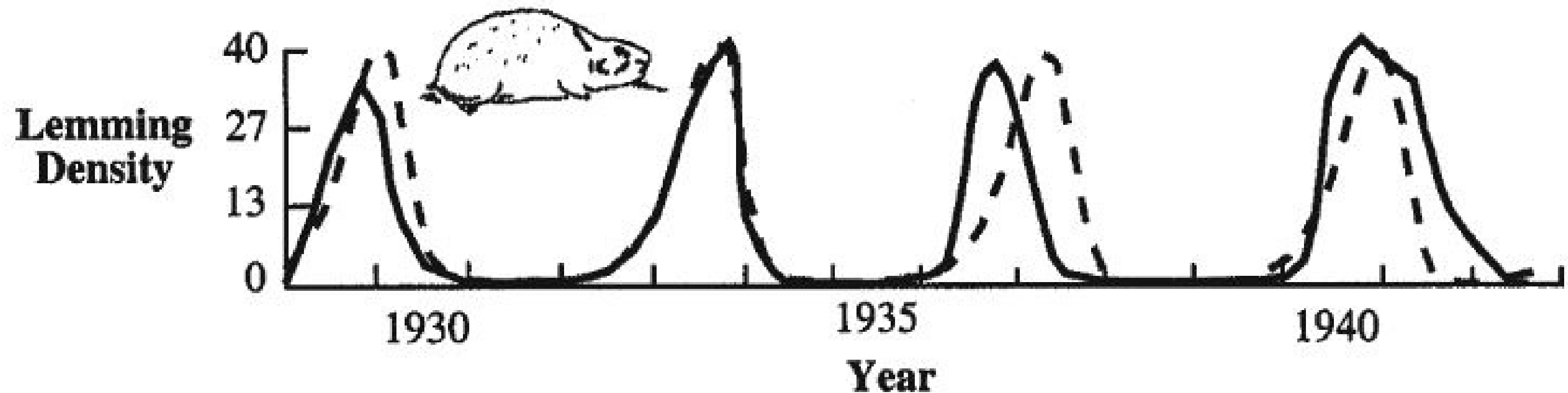
*Figura retirada de tese de doutorado de Maria Barbarossa

Population with oscillating behavior

- Blowfly *Lucilia cuprina*

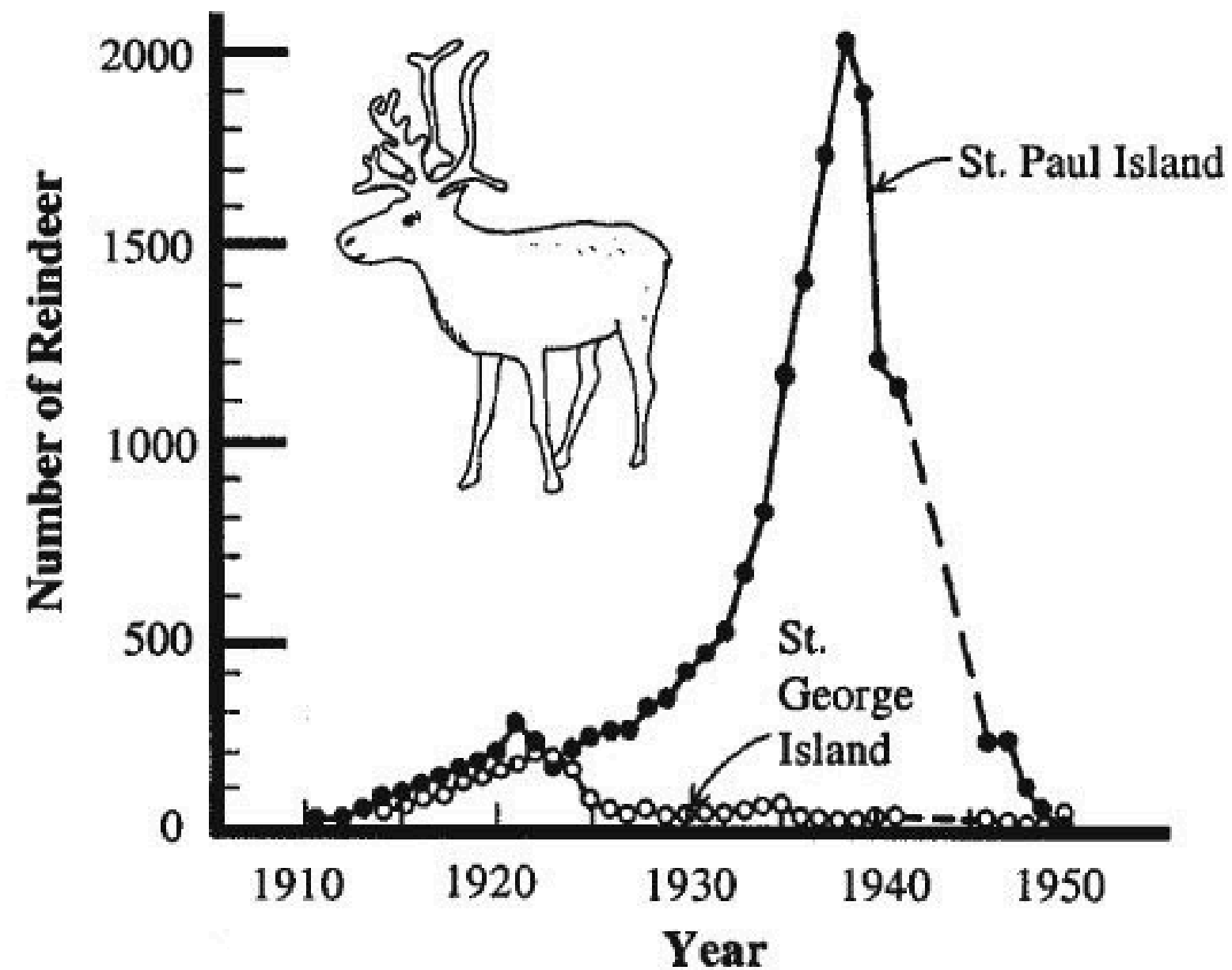


Population with oscillating behavior



*Picture from T. Erneux, Applied delay differential equations, volume 3 of Surveys and Tutorials in the Applied Mathematical Sciences. Springer, New York, 2009

Population with oscillating behavior



*Picture from T. Erneux, Applied delay differential equations, volume 3 of Surveys and Tutorials in the Applied Mathematical Sciences. Springer, New York, 2009

"It was observed that for Antarctic whale and seal populations, the length of time to maturity is a function of the amount of food (mostly krill) available. Prior to World War II, it was observed that individual seals took five years to mature, small whales took seven to ten years, and large whale species took twelve to fifteen years to reach maturity. Subsequent to the introduction of factory ships after the war, and with it a depletion of the large whale populations, there was an increase in the krill available for the seals and the remaining whales. It was then noted that seals took three to four years to mature and small whales now only took five years."

Aiello, Freedman & Wu 1992

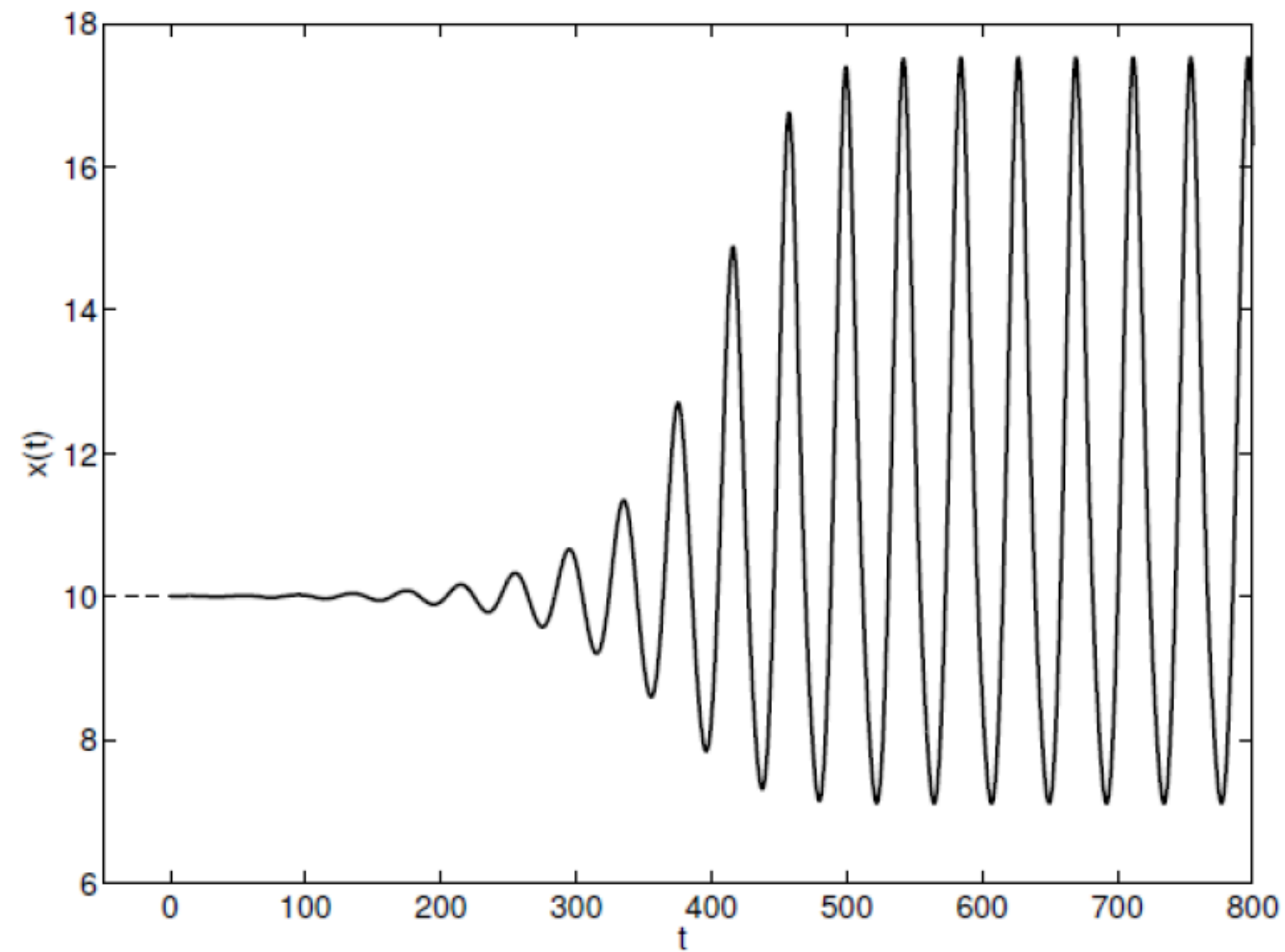


Population model with state-dependent delays

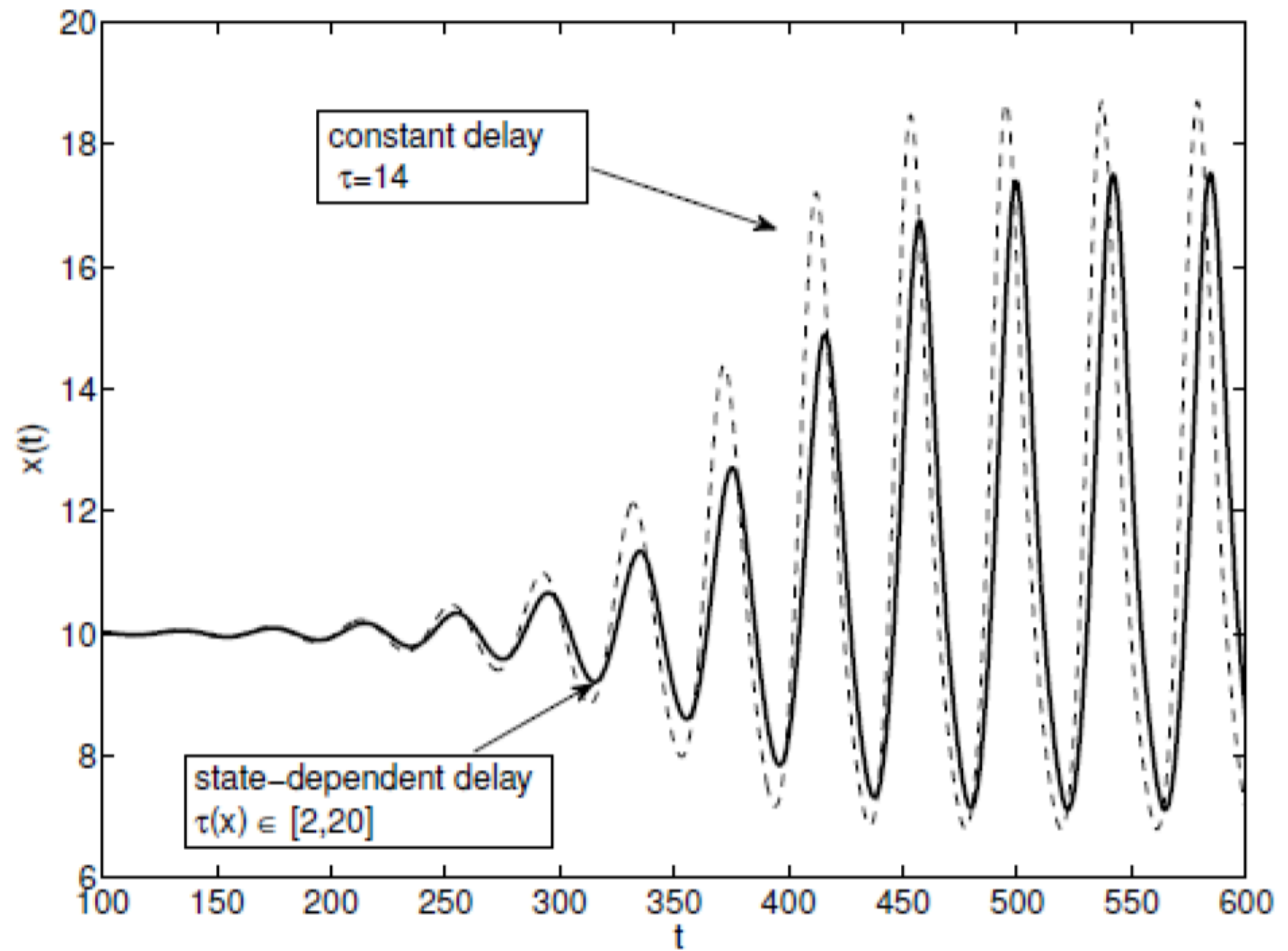
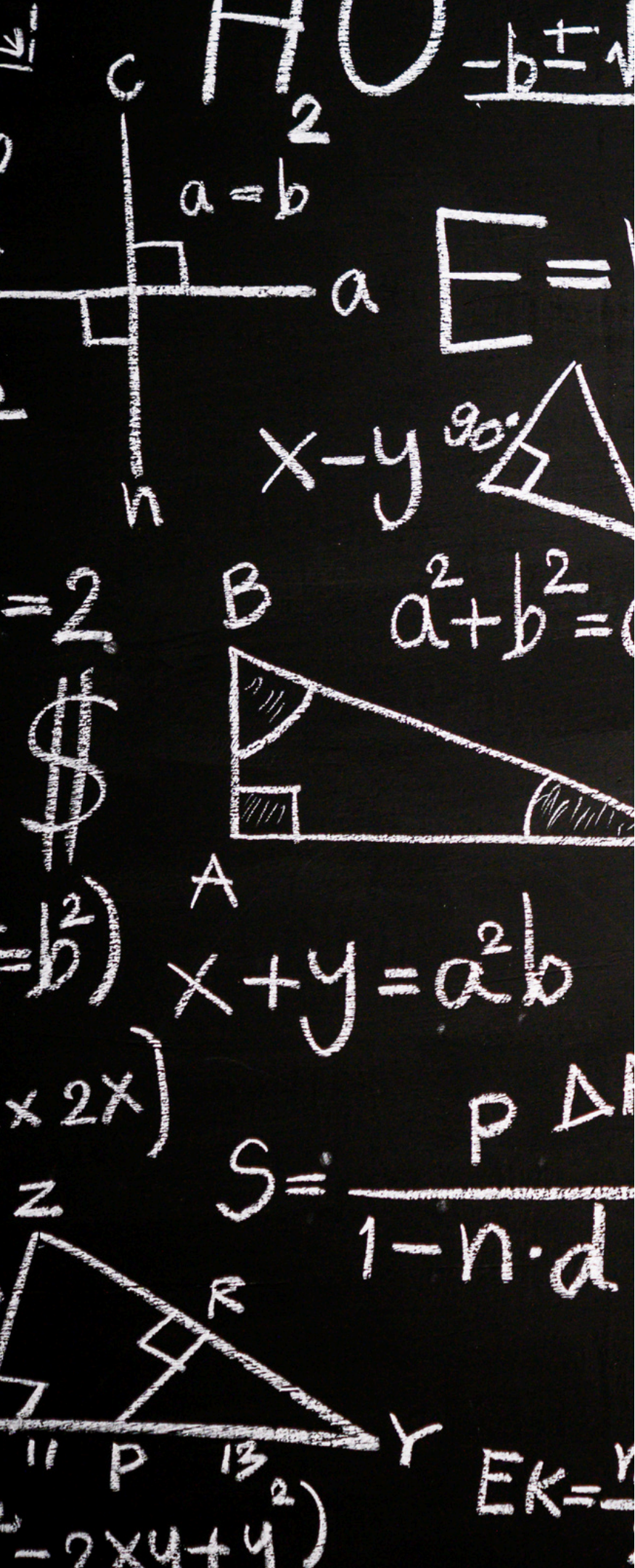
$$\dot{x}(t) = \frac{\tilde{b}_1(x(t - \tau(x(t))))e^{-\mu_0\tau(x(t))} - \tilde{\mu}_1(x(t))x(t)}{1 + \dot{\tau}(x(t))\tilde{b}_1(x(t - \tau(x(t))))e^{-\mu_0\tau(x(t))}}$$

Aiello, W. G., Freedman, H. I., and Wu, J. *Analysis of a model representing stage-structured population growth with state-dependent time delay*, SIAM Journal on Applied Mathematics 52(3), pp. 855–869, 1992.

State-dependent delays $\tau(x) \in [2, 20]$



*Figura retirada de tese de doutorado de Maria Barbarossa



*Figura retirada de tese de doutorado de Maria Barbarossa

Applications

Balázs and Krisztin

Processing data

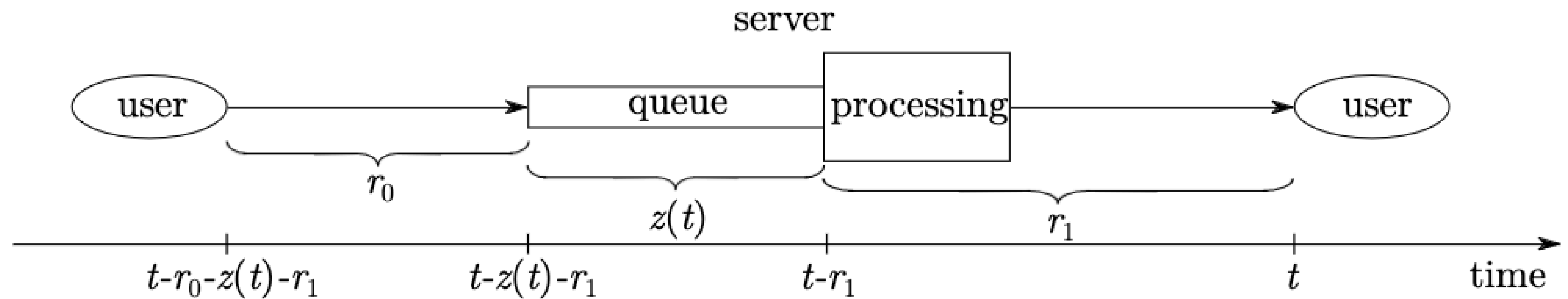


FIG. 1. *The process in time.*

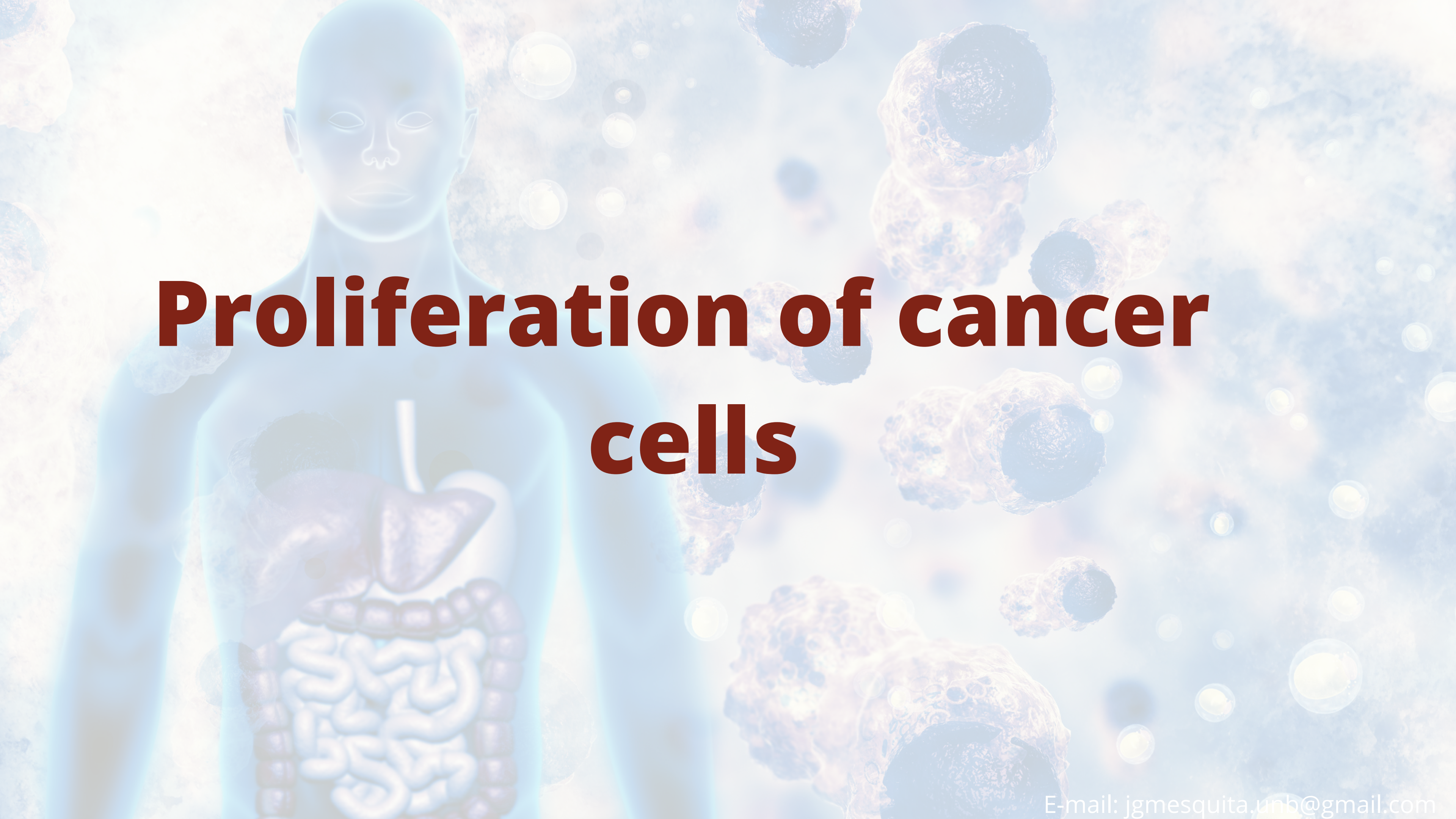
SIAM J. MATH. ANAL.
Vol. 52, No. 4, pp. 3697–3737

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A DIFFERENTIAL EQUATION WITH A STATE-DEPENDENT QUEUEING DELAY*

ISTVÁN BALÁZS[†] AND TIBOR KRISZTIN[‡]





Proliferation of cancer cells

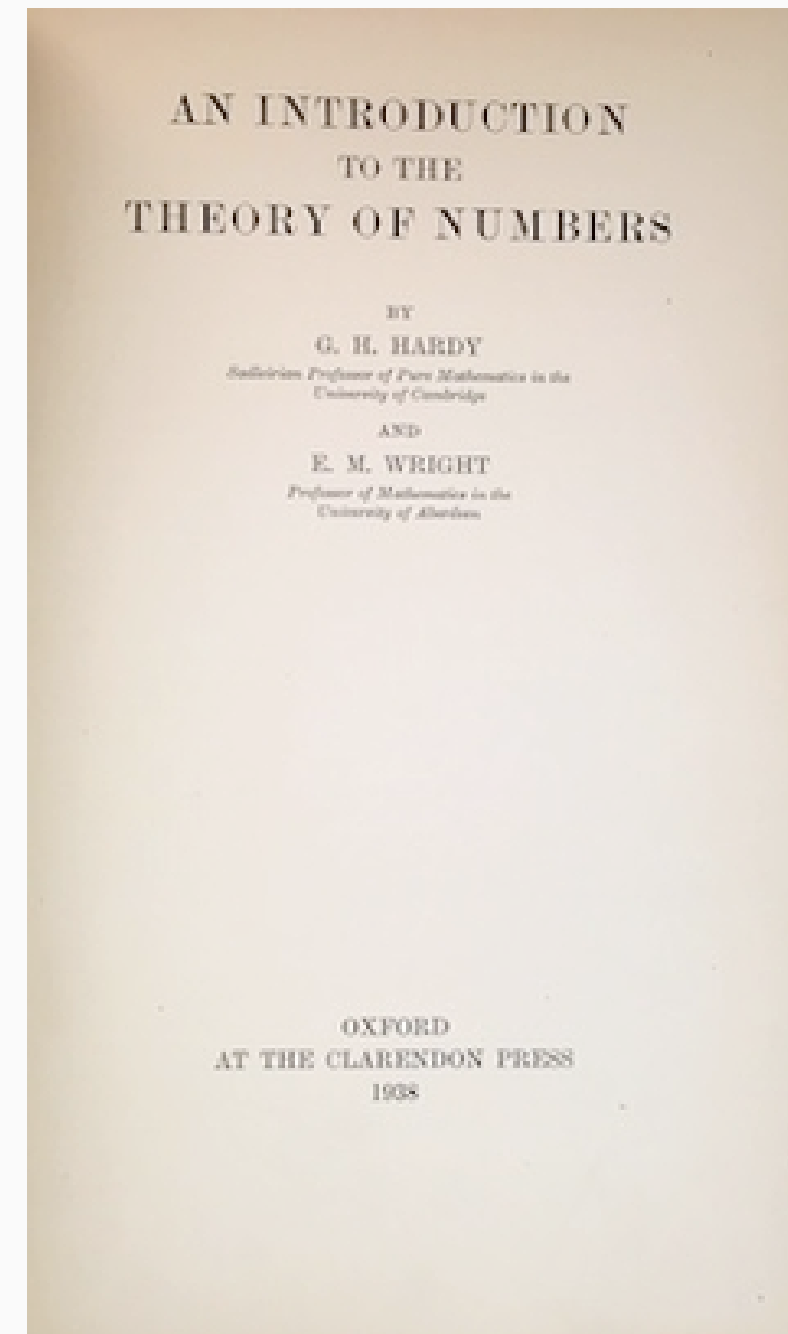
Rodney Driver

Two-body problems in eletrodynamics



E. Wright

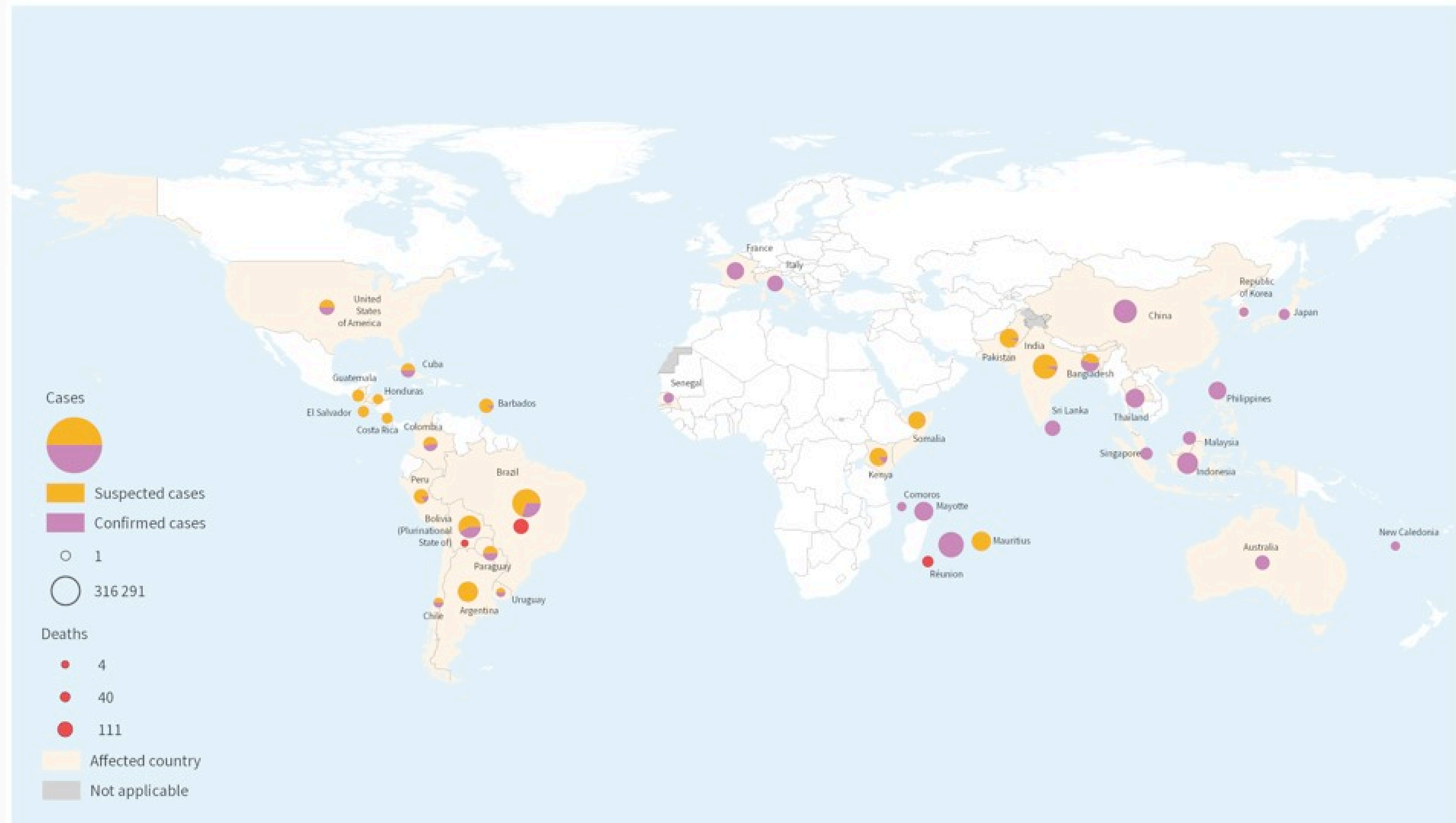
***Connections between number theory and
delay equations***



Epidemiology Model



Spread of CHIKV



The designations employed and the presentation of the material in this publication do not imply the expression of any opinion whatsoever on the part of WHO concerning the legal status of any country, territory, city or area or of its authorities, or concerning the delimitation of its frontiers or boundaries. Dotted and dashed lines on maps represent approximate border lines for which there may not yet be full agreement.

Data Source: World Health Organization, publicly shared data by Ministries of Health
Map Production: WHO Health Emergencies Programme
Map Date: 14 November 2025

0 1,500 3,000 Km



SYMPTOMS OF CHIKUNGUNYA



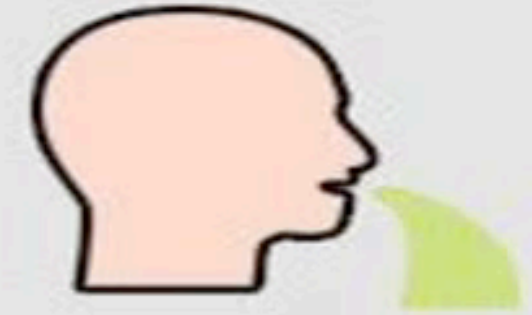
Rash



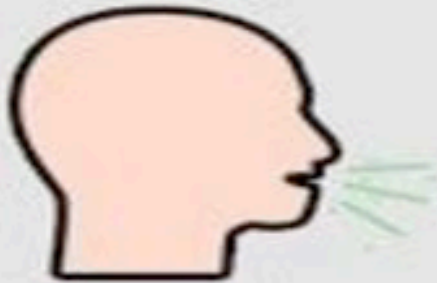
Back Pain



Joint Pain



Vomiting



Nausea



Headache



Chills

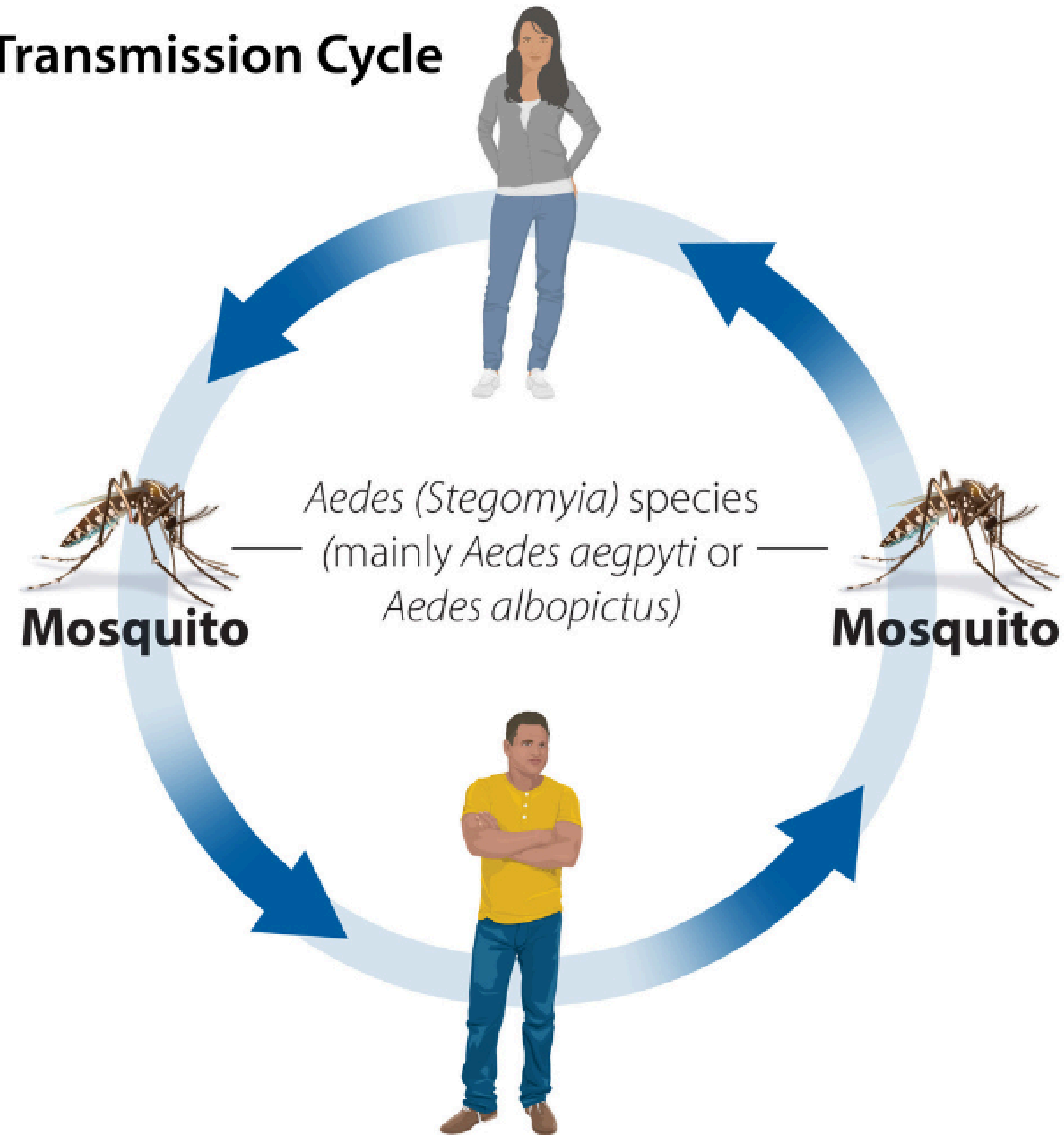


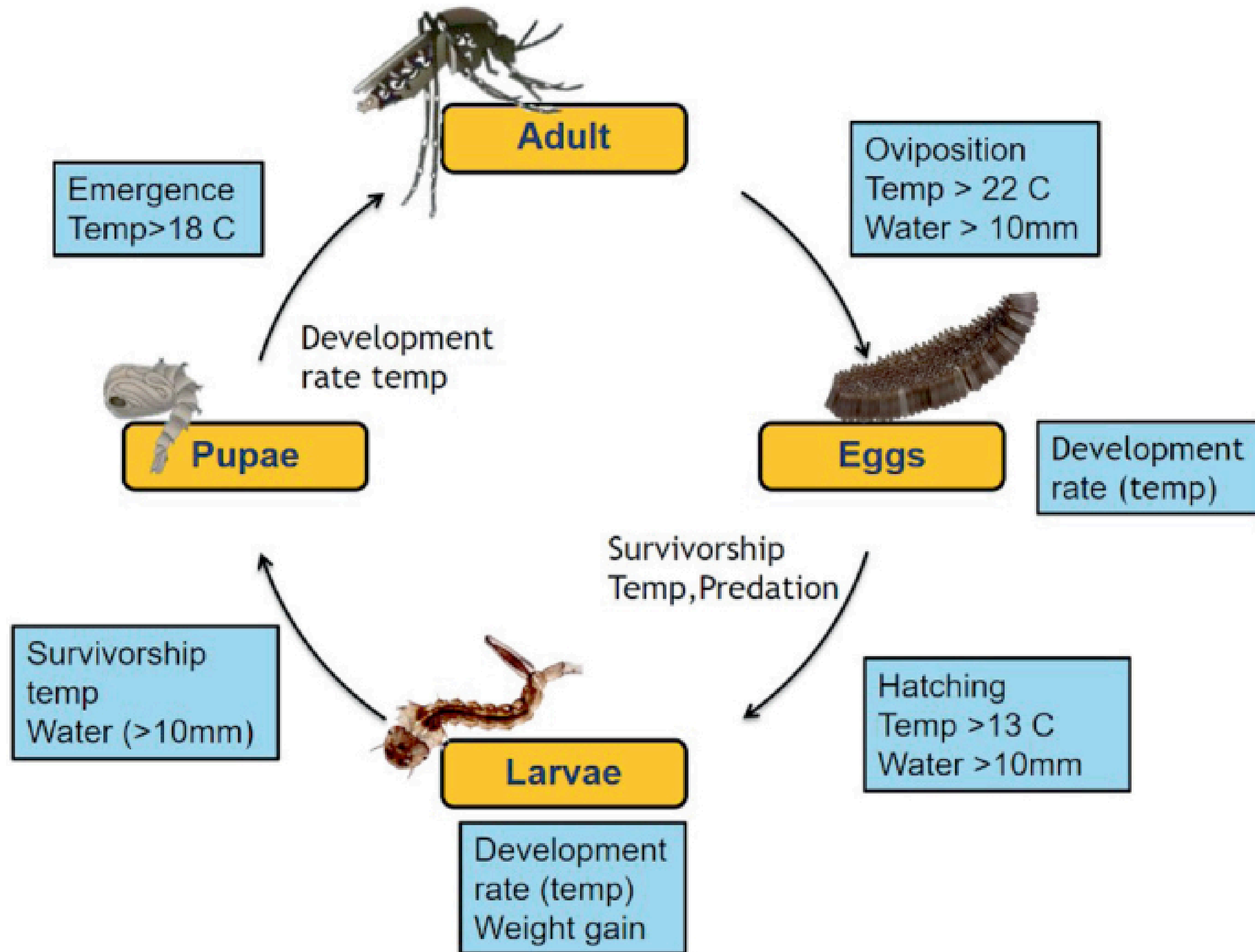
Fever

Symptoms



Chikungunya Virus Transmission Cycle





Let τ_h , $\tau^i(t)$, $\tau_l^i(t)$ represent the intrinsic incubation period, extrinsic incubation period and the larvae development period, respectively.

- ▶ $S_h^i(t)$: number of susceptible
- ▶ $E_h^i(t)$: number of exposed
- ▶ $I_h^i(t)$: number of infected
- ▶ $A_h^i(t)$: number of asymptomatic
- ▶ $R_h^i(t)$: number of recovered

humans in patch i ($i = 1, 2, \dots, n$) at time t , respectively.

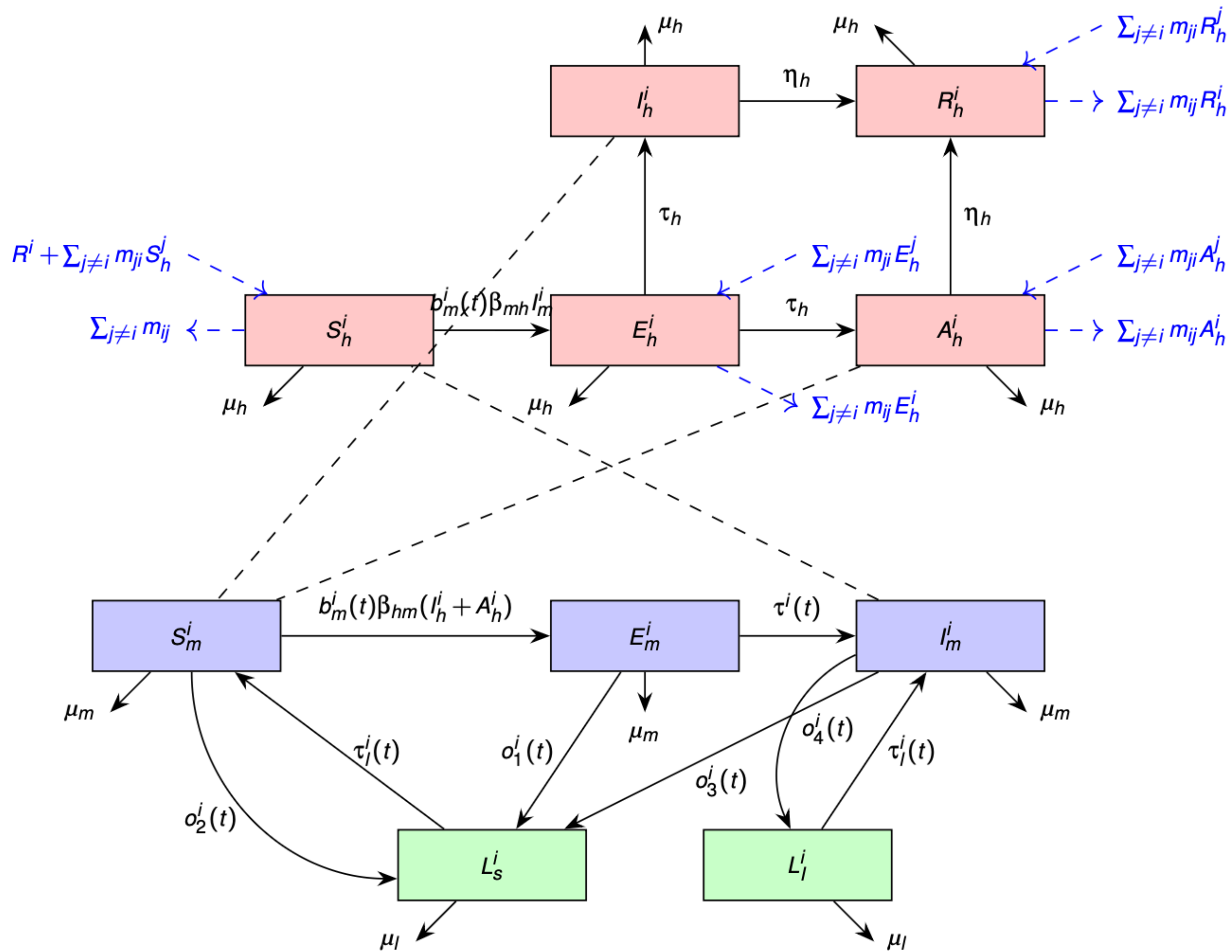
- ▶ $S_m^i(t)$: number of susceptible
- ▶ $E_m^i(t)$: number of exposed
- ▶ $I_m^i(t)$: number of infected

mosquitoes in patch i ($i = 1, 2, \dots, n$) at time t , respectively.

- ▶ $L_s^i(t)$: number of susceptible
- ▶ $L_i^i(t)$: number of infected

larvae in patch i ($i = 1, 2, \dots, n$) at time t , respectively.

R^i	Human population recruitment rate in patch i
μ_h	Human natural death rate
$\mu_m^i(t)$	Mosquito natural death rate in patch i
$b_m^i(t)$	Per capita mosquito biting rate in patch i
β_{mh}	Human-mosquito transmission probability
β_{hm}	Mosquito-human transmission probability
$\tau^i(t)$	The extrinsic incubation period in mosquitoes in patch i
$\tau_l^i(t)$	The mean development stage time in larvae in patch i
τ_h	The intrinsic incubation period in humans
$\mu_b^i(t)$	Larvae birth rate in patch i
a	Proportion of asymptomatic infectious humans
p	Vertical transmission probability
$K^i(t)$	Carrying capacity for mosquitoes in patch i
$\mu_l^i(t)$	Larvae death rate in patch i
m_{ij}	Traveling rate from patch i to patch j



$$\frac{dS_h^i}{dt} = R^i - b_m^i(t)\beta_{mh}S_h^i(t)I_m^i(t) - \mu_h S_h^i(t) + \sum_{j \neq i} m_{ji}S_h^j(t) - \sum_{j \neq i} m_{ij}S_h^i(t)$$

$$\begin{aligned} \frac{dE_h^i}{dt} = & b_m^i(t)\beta_{mh}S_h^i(t)I_m^i(t) - b_m^i(t - \tau_h)\beta_{mh}(t - \tau_h)S_h^i(t - \tau_h)I_m^i(t - \tau_h)e^{-\mu_h\tau_h} \\ & - \mu_h E_h^i(t) + \sum_j m_{ji}E_h^j(t) - \sum_j m_{ij}E_h^i(t) \end{aligned}$$

$$\frac{dI_h^i}{dt} = (1 - a)\beta_{mh}b_m^i(t - \tau_h)S_h^i(t - \tau_h)I_m^i(t - \tau_h)e^{-\mu_h\tau_h} - (\eta_h + \mu_h)I_h^i(t)$$

$$\begin{aligned} \frac{dA_h^i}{dt} = & a\beta_{mh}b_m^i(t - \tau_h)S_h^i(t - \tau_h)I_m^i(t - \tau_h)e^{-\mu_h\tau_h} - \\ & (\eta_h + \mu_h)A_h^i(t) + \sum_{j \neq i} m_{ji}A_h^j(t) - \sum_{j \neq i} m_{ij}A_h^i(t) \end{aligned}$$

$$\frac{dR_h^i}{dt} = \eta_h(I_h^i(t) + A_h^i(t)) - \mu_h R_h^i(t) + \sum_j m_{ji} R_h^j(t) - \sum_j m_{ij} R_h^i(t)$$

$$\frac{dL_l^i}{dt} = \mu_b(t) \left(1 - \frac{L_s^i(t) + L_l^i(t)}{K^i(t)} \right) \rho I_m^i(t) - \mu_l^i(t) L_l^i(t)$$

$$\frac{dL_s^i}{dt} = \mu_b(t) \left(1 - \frac{L_s^i(t) + L_l^i(t)}{K^i(t)} \right) (S_m^i(t) + E_m^i(t) + (1 - \rho) I_m^i(t)) - \mu_l^i(t) L_s^i(t)$$

$$\begin{aligned} \frac{dS_m^i}{dt} = & (1 - \dot{\tau}_l^i(t)) \mu_b(t - \tau_l^i(t)) \left(1 - \frac{L_s^i(t - \tau_l^i(t)) + L_l^i(t - \tau_l^i(t))}{K^i(t - \tau_l^i(t))} \right) \\ & \times (S_m^i(t - \tau_l^i(t)) + E_m^i(t - \tau_l^i(t)) + (1 - \rho) I_m^i(t - \tau_l^i(t))) e^{-\int_{t-\tau_l^i(t)}^t \mu_l^i(s) ds} \\ & - b_m^i(t) \beta_{hm} S_m^i(t) (I_h^i(t) + A_h^i(t)) - \mu_m^i(t) S_m^i(t) \end{aligned}$$

$$\begin{aligned}
\frac{dE_m^i}{dt} &= b_m^i(t) \beta_{hm} S_m^i(t) (I_h^i(t) + A_h^i(t)) - \mu_m^i(t) E_m^i(t) - \\
&\quad (1 - \dot{\tau}^i(t)) \beta_{hm} b_m^i(t - \tau^i(t)) (I_h^i(t - \tau^i(t)) + A_h^i(t - \tau^i(t))) S_m^i(t - \tau^i(t)) e^{-\int_{t-\tau^i(t)}^t \mu_m^i(s) ds} \\
\frac{dI_m^i}{dt} &= \left(1 - \dot{\tau}_l^i(t)\right) \mu_b(t - \tau_l^i(t)) \left(1 - \frac{L_l^i(t - \tau_l^i(t)) + L_s^i(t - \tau_l^i(t))}{K^i(t - \tau_l^i(t))}\right) \\
&\quad p I_m^i(t - \tau_l^i(t)) e^{-\int_{t-\tau_l^i(t)}^t \mu_l^i(s) ds} - \mu_m^i(t) I_m^i(t) + \\
&\quad + \left(1 - \dot{\tau}^i(t)\right) \beta_{hm} b_m^i(t - \tau^i(t)) I_h^i(t - \tau^i(t)) S_m^i(t - \tau^i(t)) e^{-\int_{t-\tau^i(t)}^t \mu_m^i(s) ds} \\
&\quad + \left(1 - \dot{\tau}^i(t)\right) \beta_{hm} b_m^i(t - \tau^i(t)) A_h^i(t - \tau^i(t)) S_m^i(t - \tau^i(t)) e^{-\int_{t-\tau^i(t)}^t \mu_m^i(s) ds}
\end{aligned}$$

with initial value conditions

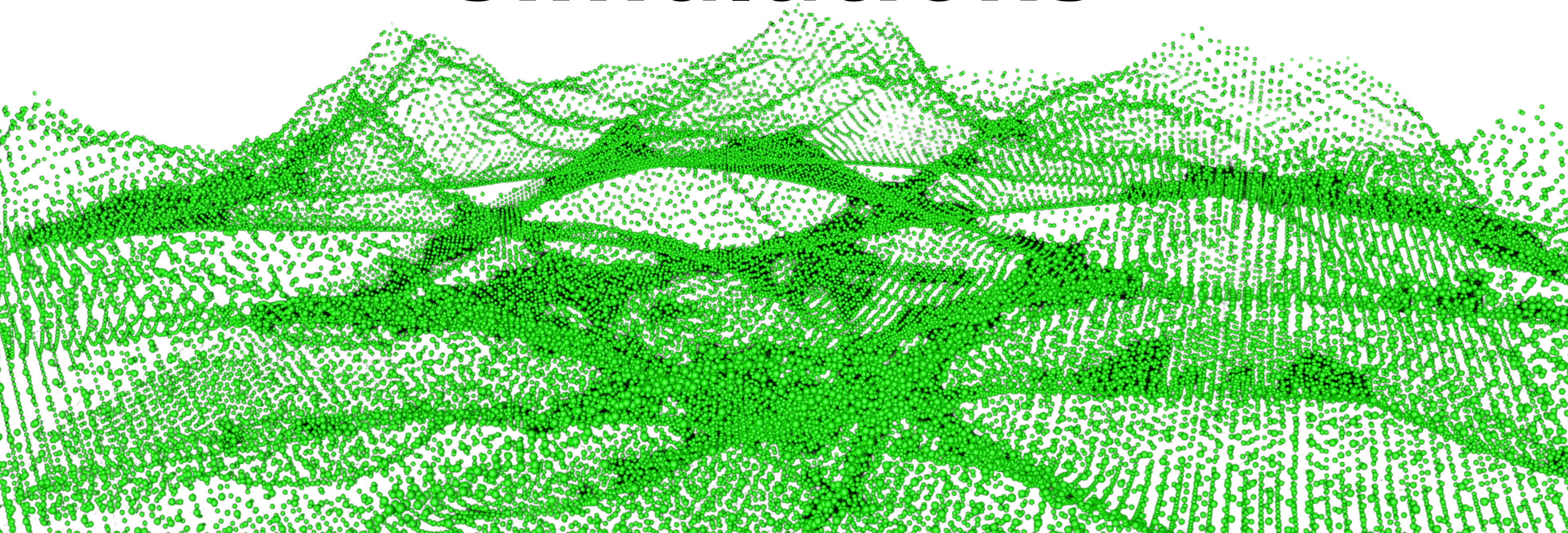
$$\begin{aligned} S_h^i(s) &= \varphi_1^i(s), \quad E_h^i(0) = \varphi_2^i(0), \quad I_h^i(s) = \varphi_3^i(s), \\ A_h^i(s) &= \varphi_4^i(s), \quad R_h^i(0) = \varphi_5^i(0), \\ L_l^i(s) &= \varphi_6^i(s), \quad L_s^i(s) = \varphi_7^i(s), \quad S_m^i(s) = \varphi_8^i(s), \\ E_m^i(s) &= \varphi_9^i(s), \quad I_m^i(s) = \varphi_{10}^i(s), \end{aligned} \tag{4}$$

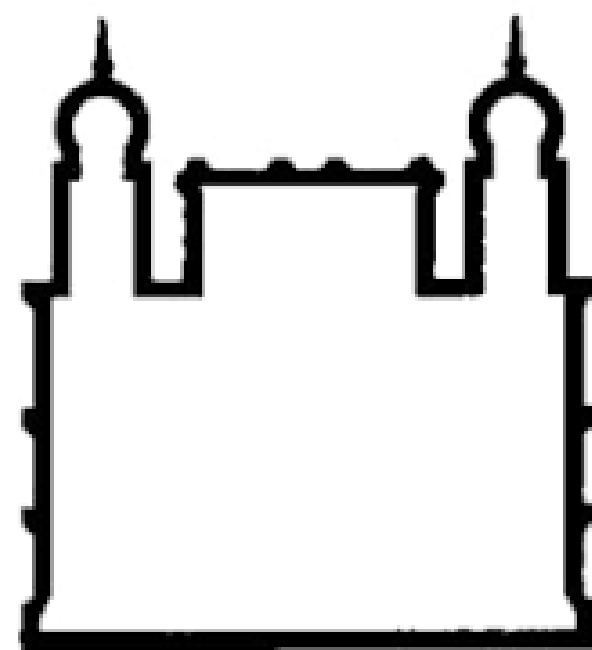
for $s \in [-\tilde{\tau}, 0]$, in which

$$\tilde{\tau} := \max\{\tau_0^i\}_{i=1,\dots,n} \text{ where } \tau_0^i := \max\left\{ \sup_{0 \leq t \leq w} \tau^i(t), \tau_h^i, \sup_{0 \leq t \leq w} \tau_l^i(t) \right\},$$

and $\tau^i(t), \tau_l^i(t)$ are strictly positive functions.

Numerical Simulations





FIOCRUZ



Ceará
Pernambuco
Bahia
Amazonas
Pará
Rio de Janeiro
São Paulo
Distrito Federal
Minas Gerais

Parameter	Constants and Temperature Dependent Formula
μ_h	$\frac{1}{74.5 \cdot 365} \text{ day}^{-1}$
N_{h0M}	2700000
N_{h0B}	109966000
R^{BR}	$\frac{109966000}{77 \cdot 365} \text{ day}^{-1}$
R^{MIA}	$\frac{2700000}{80 \cdot 365} \text{ day}^{-1}$
$\mu_m(T)$	$0.8692 - 0.1590T + 0.01116T^2 - 0.0003408T^3 + 3.809(10^{-6})T^4 \text{ day}^{-1}$
$b_m(T)$	$\frac{(0.0943 + 0.0043 \cdot T)}{N} \text{ day}^{-1}$, N is the patch's total population
β_{mh}	0.13
β_{hm}	0.3
$\mu_l(T)$	$2.130 - 0.3797T + 0.02457T^2 - 0.0006778T^3 + 0.000006794T^4 \text{ day}^{-1}$
$\tau(T)$	$4 + e^{5.15 - 0.123 \cdot T} \text{ day}$
τ_h	5day
$\mu_b(T)$	$-0.32 + 0.11 \cdot T - 0.12 \cdot T^2 + 0.01 \cdot T^3 - 0.00015 \cdot T^4 \text{ day}^{-1}$
a	0.45

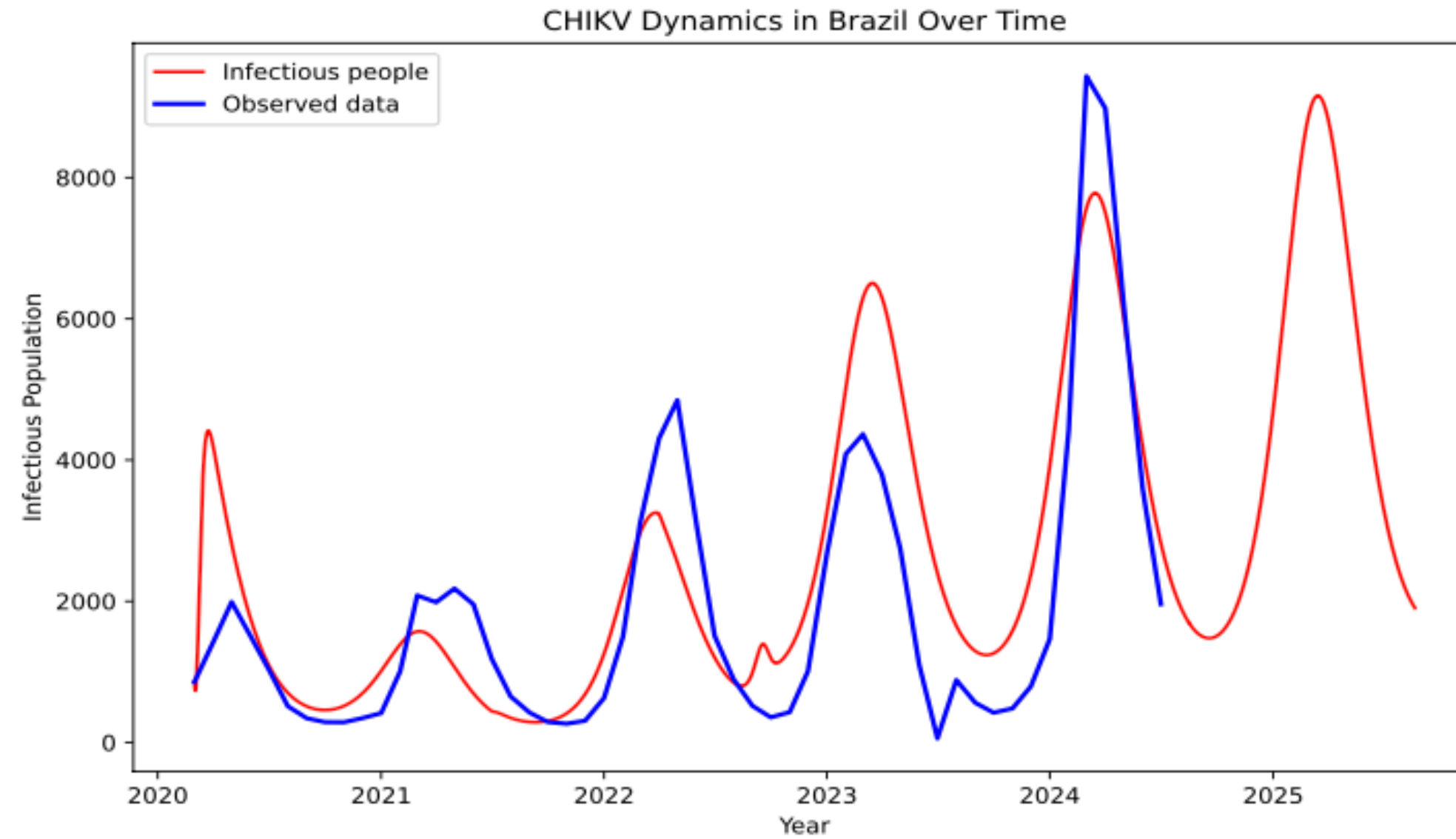
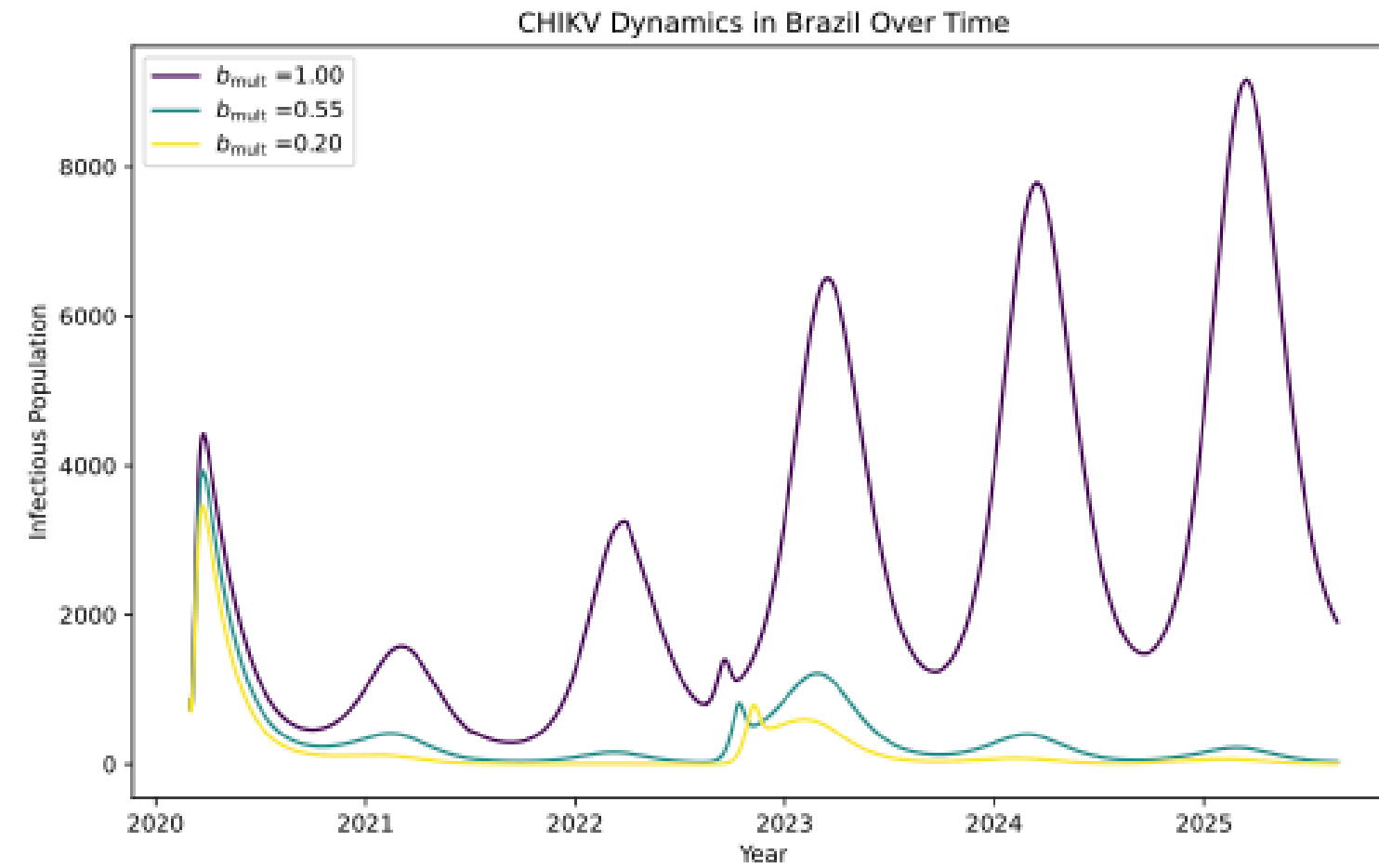
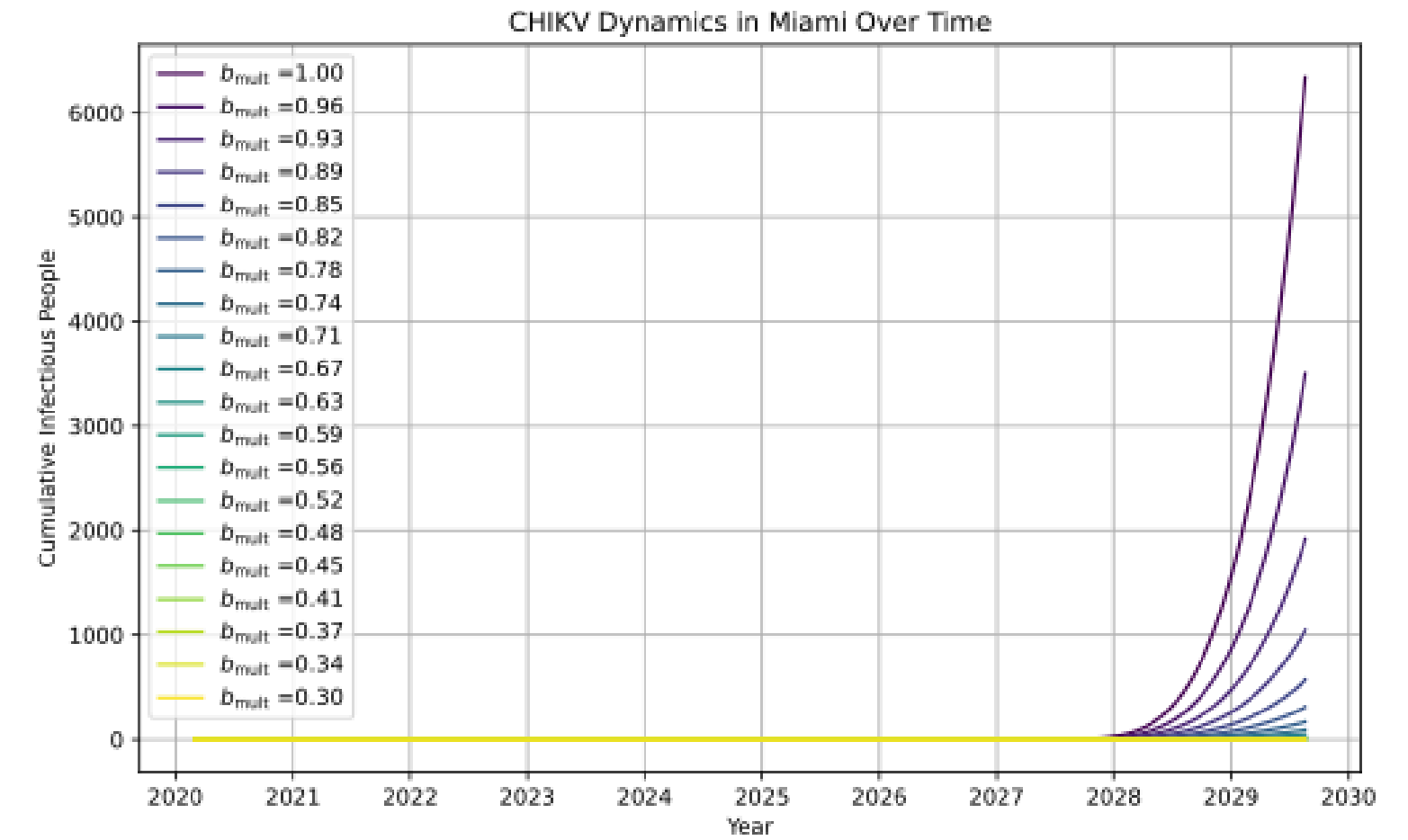


Figure 4.2: Simulation of the number of symptomatic infectious human population in Brazil from March 2020 to 2025. The blue curve represents the data and the red curve is the simulation of the model.



(a) Per capita biting rate sensitive analysis in Brazil projected for 2025



(b) Sensitive analysis of the per capita biting rate in Miami projected for 2030

Figure 4.4: Sensitive analysis of the per capita biting rates.

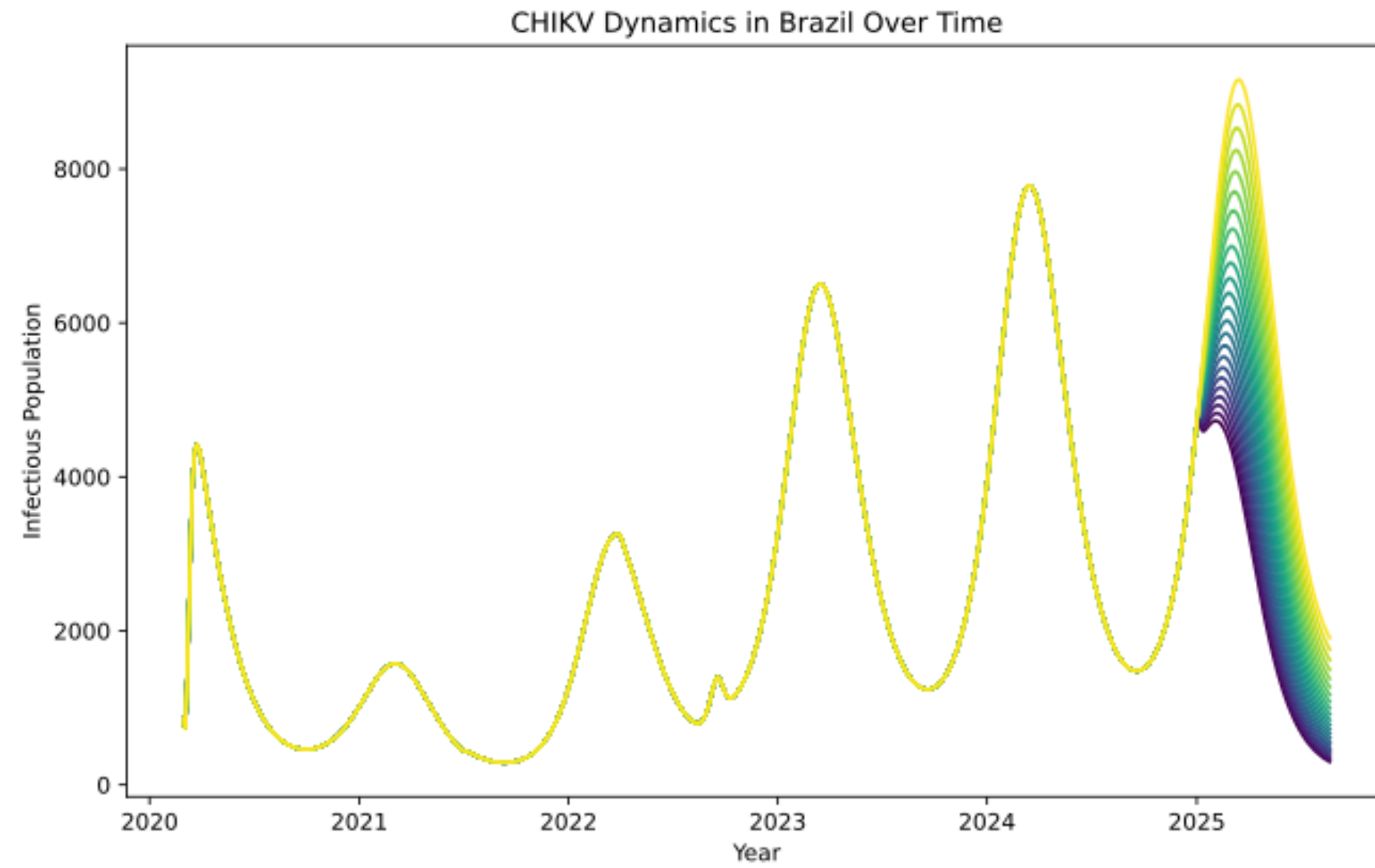
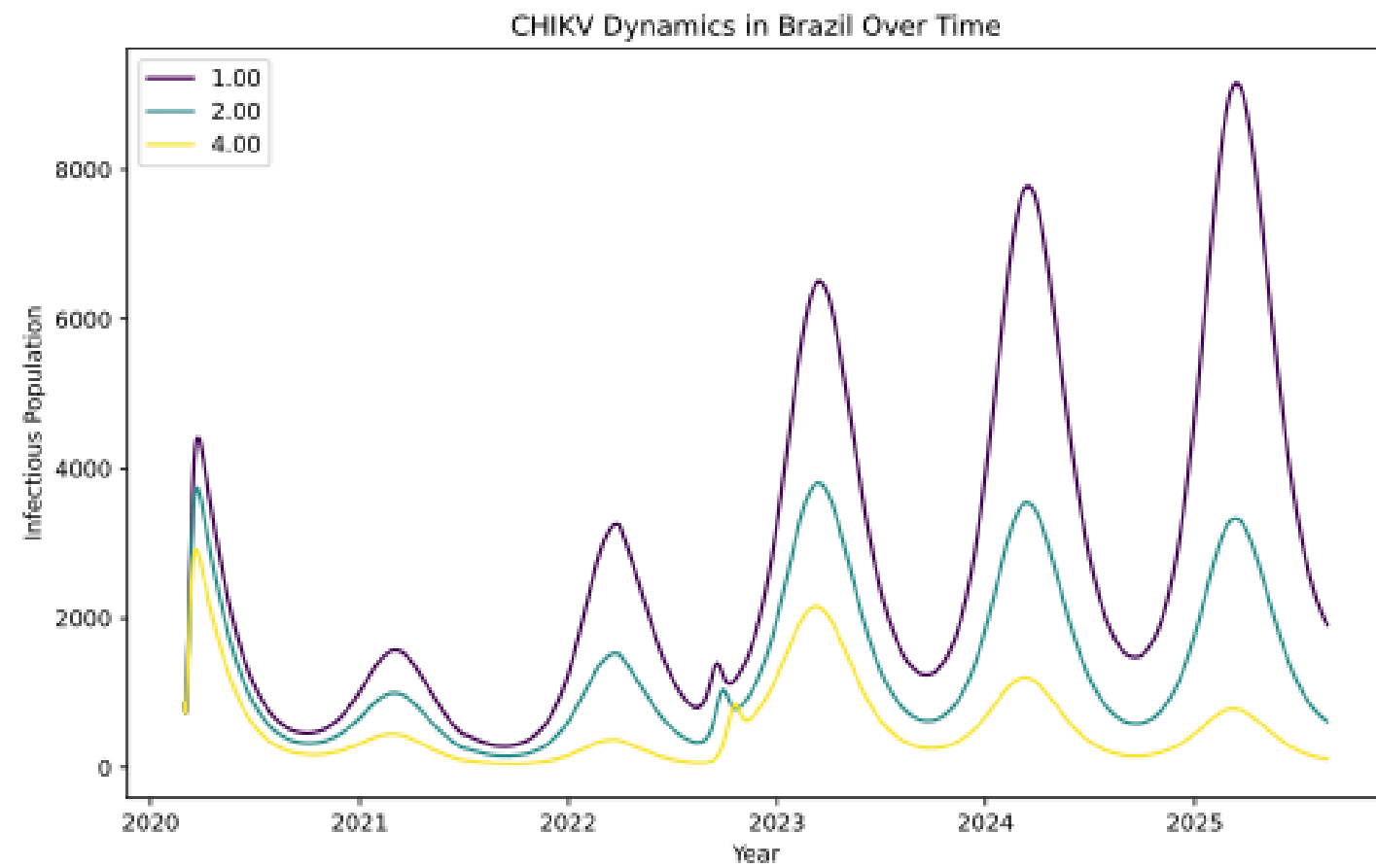
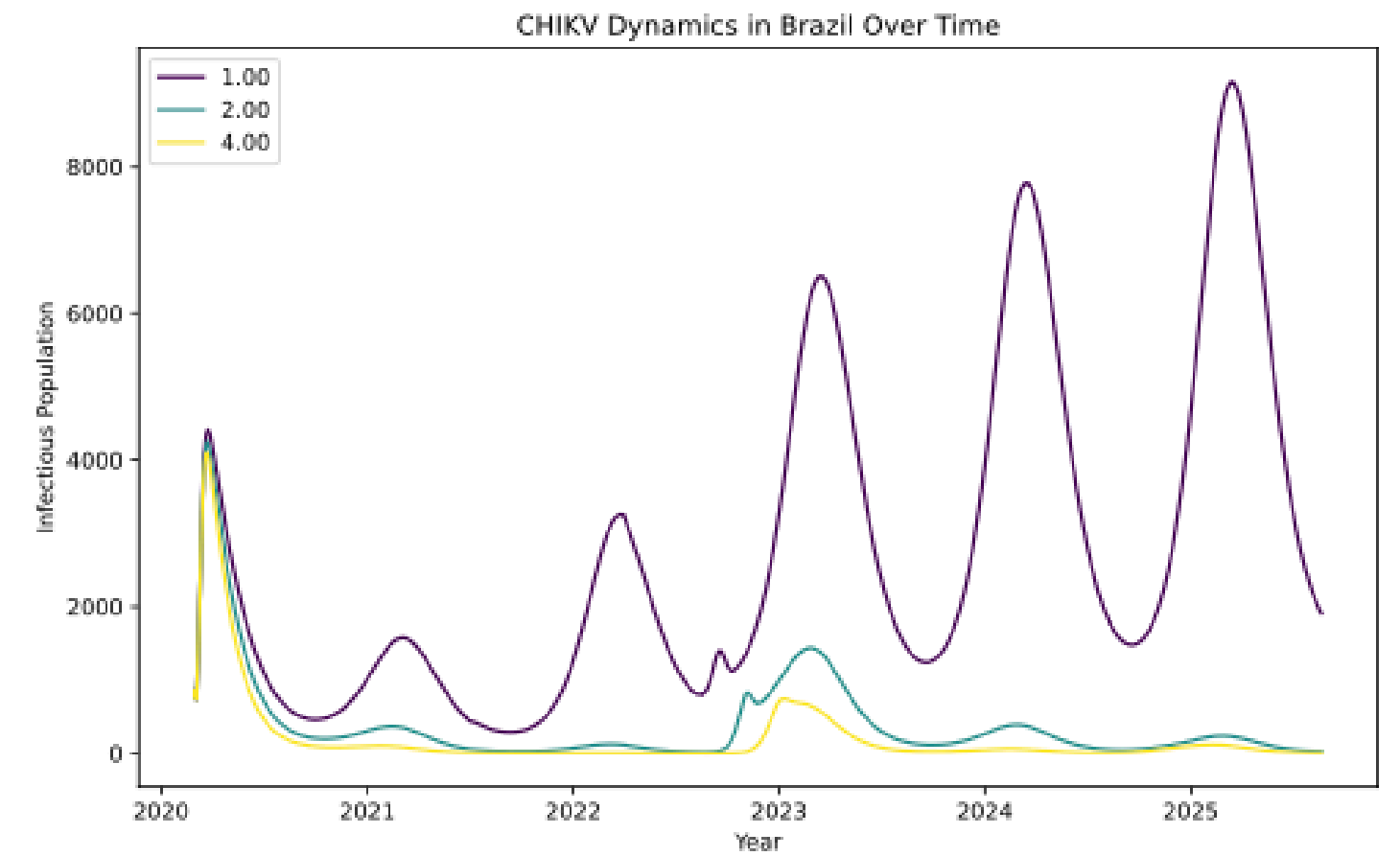


Figure 4.7: Projection of chikungunya for 2025 in Brazil with varying biting rate control measures.

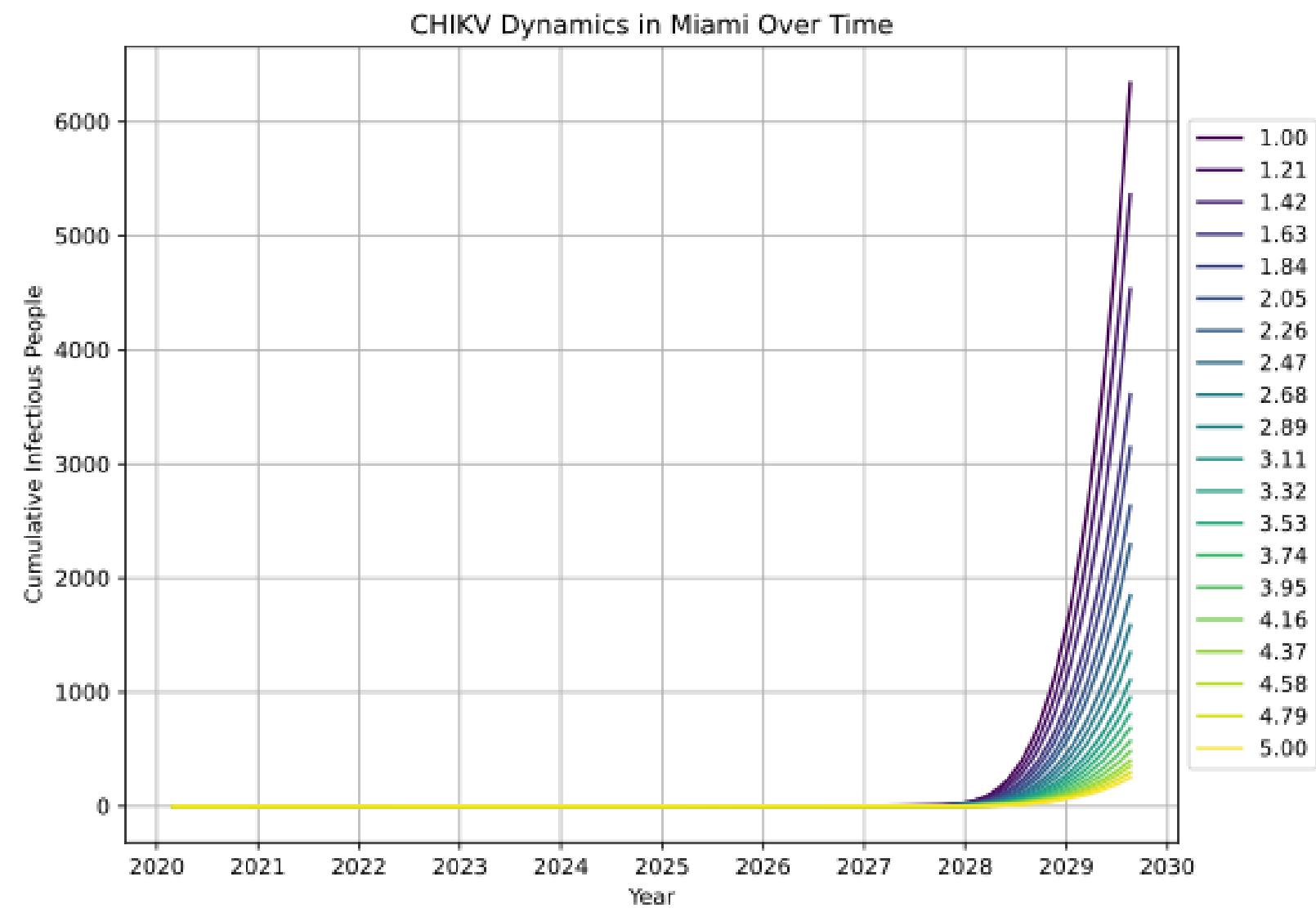


(a)

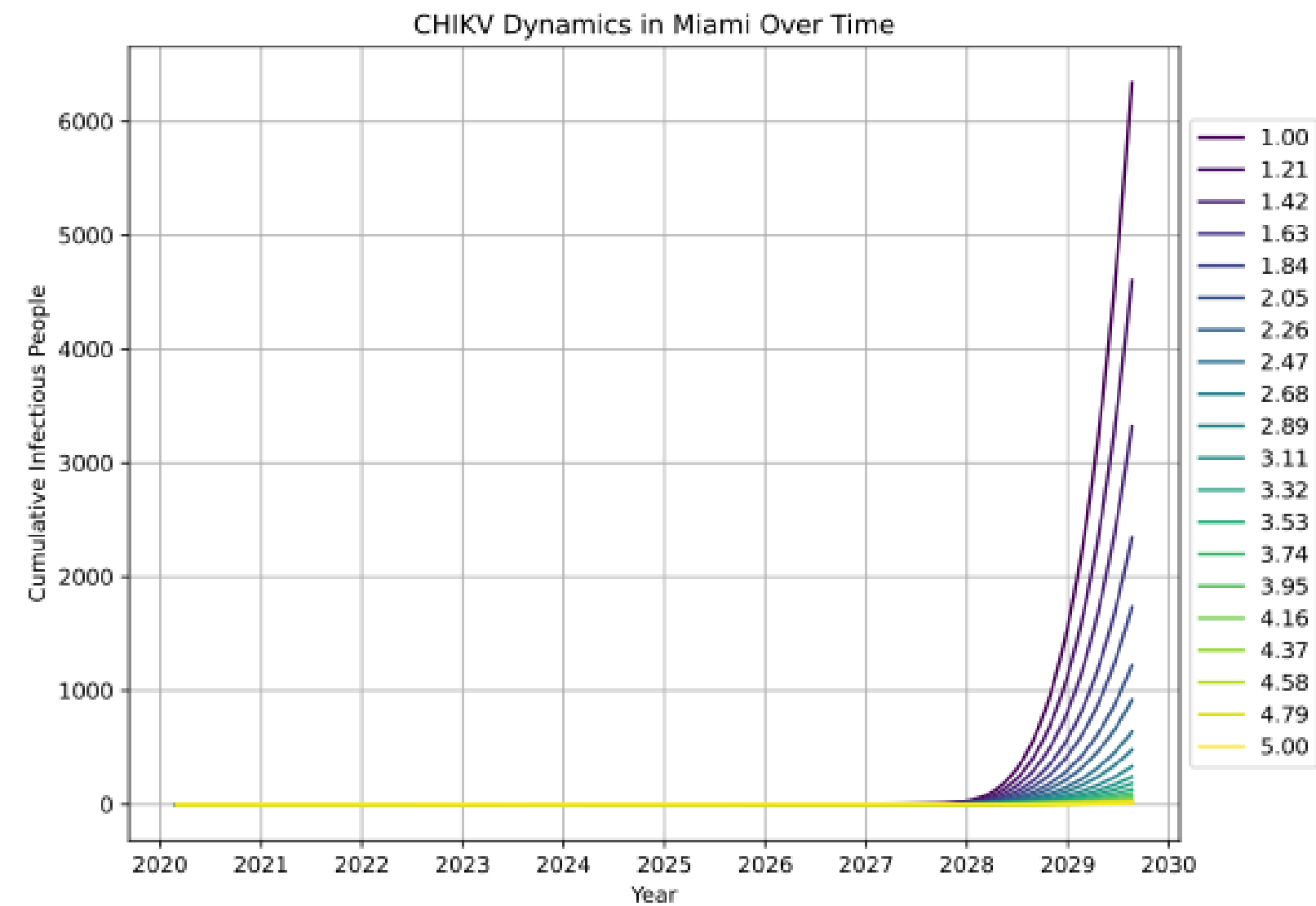


(b)

Figure 4.5: Sensitive analysis on (a) larvae and (b) mosquitoes death rate multiplier in Brazil.



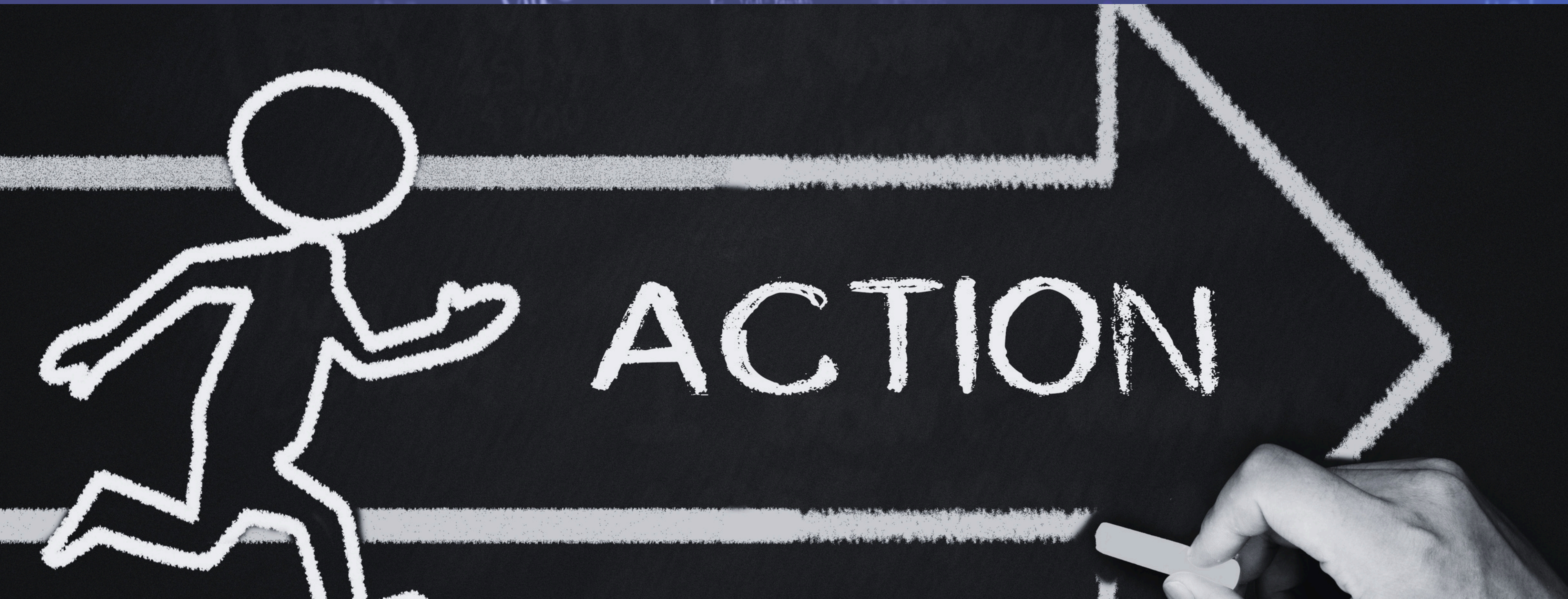
(a)



(b)

Figure 4.6: Sensitive analysis on (a) larvae and (b) mosquitoes death rate multiplier in Miami.

Open and Developing Problems



Teşekkür ederim תודה Hvala

Hvala

Ευχαριστώ

Dankon

Хвала

Gracias

Grazie

謝謝

شكرا لك

Danke

Thank you

Merci

有難く

Tack

Спасибо

Obrigado

감사합니다

Köszönöm

Dank u

Спасибі

有り難う

谢谢

Благодаря

Asante

धन्यवाद

ありがとう

Terima kasih

Mulțumesc

Dank u

شکر

Kiitos

Dziękuję