Conversations on Space Debris

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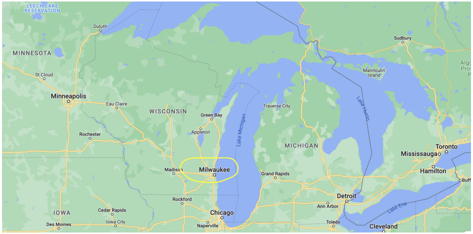
University of Łódź
October 16th, 2024

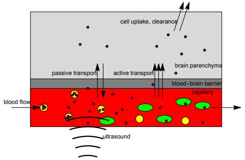
Home, sweet home

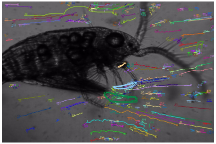


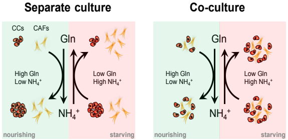
A little bit about myself

My primary research interests are application of mathematics to problems in the life sciences, including

- drug delivery to the brain using ultrasound sensitive liposomes
- behavioral ecology of zooplankton
- metabolic interactions in tumors







Collaborators

1. John Jurkiewicz, PhD in Mathematics, University of Wisconsin - Milwaukee, 2022
2. Victor Florez, undergraduate student, 2023-present

Overview of the talk

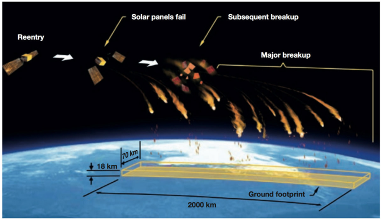
- Overview of Near-Earth environment
- The Kessler Syndrome, growth of the debris population
- Existing modeling approaches (discrete, massive ODE systems)
- Population dynamics model (reaction-diffusion PDE)
- Parametrization of our model using literature and public object catalogs
- Numerical simulations and comparison to observations
- Outlook on future research - whom do we have "conversations" with?

Near-Earth Orbit

- Portion of outer space in proximity to Earth
- Classically divided into
  - Low-Earth Orbit (LEO, altitude $\leq$ 2,000 km)
  - Medium-Earth Orbit (MEO, 2,000 - 36,000 km)
  - Geosynchronous Orbit (GEO, $\geq$ 36,000 km)
- The LEO region is host to the majority of man-made satellites.

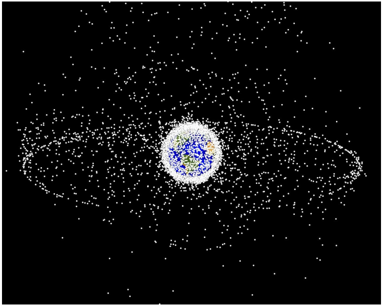
Low-Earth Orbit Debris Population

- Since Sputnik 1 (1957), huge numbers of satellites have been launched, many of whom have been deactivated, lost parts, and others have collided.
- The number of objects $\geq$ 1 mm in diameter today is estimated in the millions.
- The orbital velocities are ≈ 10 km s^{-1}; high danger if objects collide, e.g. Iridium 33 - Kosmos 2251 (2009)
- Other sources of debris: intentional destructions, e.g. Fengyun 1C (2007), Kosmos 1408 (2021), explosion of rocket stages.



World Economic Forum; weforum.org

Near-Earth Orbit Population



Computer-generated image of objects in Near-Earth Orbit. Clearly visible is the LEO "cloud" and the GEO (equatorial) "ring". From NASA's Orbital Debris Program Office; available at orbitaldebris.jsc.nasa.gov.

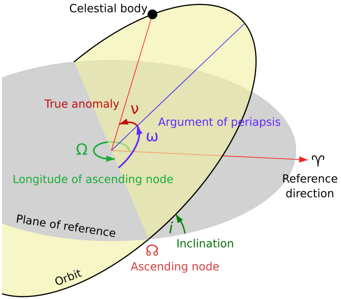
Kessler Syndrome

- Possibility of rampant, chaotic growth of population due to inter-object collisions (mathematicians: think of $x' = x^2$!).
- First discussed by Kessler and Cour-Palais, 1978
- Obvious item of concern for policymakers

We construct a computationally feasible model for temporal evolution of the space debris population and parametrize it with observation data from space debris catalogs.

Current approaches to modeling

- The standard is a discrete, ODE-based approach.
- Each item in orbit is defined by 6 **orbital elements** describing its position and velocity
- These $10^4 - 10^6$ orbital elements are propagated forward in time to determine the state up to 100 years into the future.

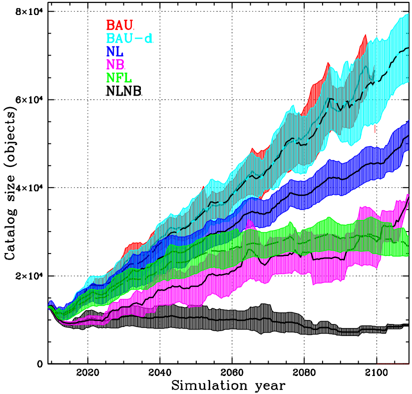


Elements to describe a Kepler orbit. From wiki/Orbital_elements

Current approaches to modeling: discrete

- Nikolaev, et. al., 2012: incorporates explosions and collisions
- Computes orbital trajectories for every observed object in orbit
- If a collision is detected, new debris generated
- Expanded catalog propagated, process repeats
- Accurate, but requires 10^4 - 10^6 individual propagations

Current approaches to modeling: discrete



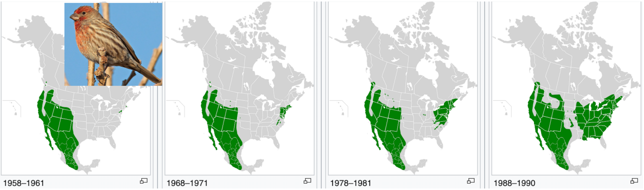
Simulated growth of the catalog size of known objects under various mitigation strategies up to the year 2110. The "business as usual" (BAU, top funnel) is the most pessimistic scenario. From Nikolaev *et al.*, 2012.

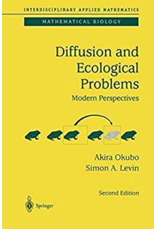
Current approaches to modeling: continuous

- Continuum approach pioneered by McInnes, 1993
- PDE derived via Boltzmann's Equation including drift and collision (Smoluchowski-type) terms
- Solves for volumetric density of space debris
- Computational savings compared to the discrete approach

The similarity to population dynamics

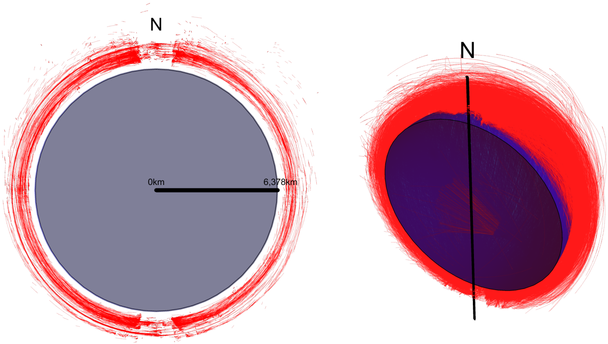
Why do I present this at a conference called “Biomath”?





(Left) Spread of the introduced House Finch *(Haemorhous mexicanus)* in Eastern North America, 1950-1990, where it competes with the native Purple Finch *(H. purpureus)*. From wikipedia.org/wiki/House_finch
(Right) Book by Akira Okubo, 1980 & 2001.

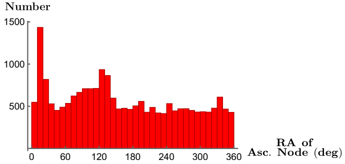
Assumptions for model building



Distribution of orbits in LEO in March 2022, data from US Space Command.

- We assume spherical symmetry of the population density.
- Earth is a perfect sphere with equatorial radius $r_E = 6378$ km.

Assumptions for model building



The right ascension (longitude) of ascending node (the longitude at which the orbit intersects Earth's orbital plane) as of March 2nd, 2022 is approximately uniformly distributed.

Population dynamics model

We propose a spherically-symmetric diffusion-collision equation with birth term for the evolution of the debris density. Details of the model are

- a radially-dependent diffusivity which captures effects of solar wind and atmospheric drift,
- the "birth term" $\Lambda(r, t)$ accounts for ground-launch policy,
- homogeneous Dirichlet (lower) and homogeneous Neumann (upper) boundary conditions.

Population dynamics model

$$\frac{\partial}{\partial t}u(r,t) = \underbrace{\frac{1}{r^2}\frac{\partial}{\partial r}\left(D(r)r^2\frac{\partial}{\partial r}u(r,t)\right)}_{\text{spherical Laplacian}}$$

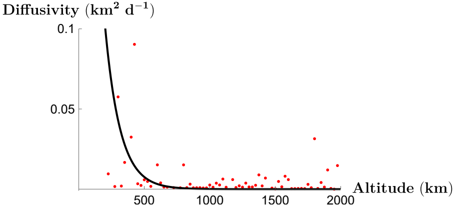
$$\underbrace{+\frac{1}{\sqrt{2}}\beta\gamma\sqrt{\frac{GM}{r+r_E}}u^2(r,t)}_{\text{growth due to binary collisions}}$$

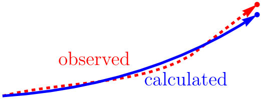
$$\underbrace{+\Lambda(r,t)}_{\text{rate of debris deposition}}$$

Collision term from McInnes, 1993; G is the gravitational constant, M is the mass of the Earth.

Diffusion rate

$$D(r) = \begin{cases} \alpha \exp(-\lambda(r - r_E)), & r < r_E + 1000 \\ \xi, & r \geq r_E + 1000 \end{cases}.$$





Diffusivity as function of altitude; idea from Riesing & Kahoy, 2015: propagate objects from the past to a later time and compare with their locations from a database.

github.com/jurkiew4/Space-Debris-Analytic-Model

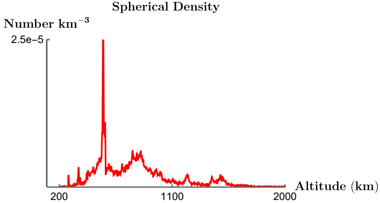
Boundary and initial conditions

$$u(r_E + 200, t) = 0,$$
$$\frac{\partial}{\partial r} u(r_E + 2000, t) = 0,$$
$$u(r, 0) = u_0(r), \quad r \geq r_E + 200.$$

- At the lower boundary we have atmospheric burn up,
- at the upper boundary we have no migration from outer space,
- the initial conditions are determined with the help from object catalogs.

Initialization of model

- "$t = 0$" are 2014, 2016, 2018, 2020 and 2022.
- The appropriate TLEs are downloaded from Space-Track (US Space Force); Space-Track.org.
- The open-source Python library `pyorbital` used to compute altitudes from TLEs.
- The cumulative density function $U(r)$ of altitudes is computed from $r_E + 200$ to $r_E + 2000$, interpolated via cubic splines, and then used to obtain $u(r, 0)$.



Parameters of the model

α	Diffusion at lowest level	$0.5783 \text{ km}^2 \text{ d}^{-1}$
ξ	Diffusion at higher altitudes	$10^{-4} \text{ km}^2 \text{ d}^{-1}$
λ	Radial decay of diffusion	0.0086 km^{-1}
β	New objects created in binary collision	2000
γ	Mean cross-sectional area of debris	$\approx 17 \text{ cm}^2$

- β based on data from known binary collisions, primarily the Iridium-Kosmos collision of 2009
- Diffusion-related parameters: exponential decay due to atmospheric effects at lower altitude
- ξ and effective altitude 1000 km set in accordance with cataloged data and Lemaitre & Hubbard, 2013
- γ used as free parameter to fit predictions to published LEGEND data.

Deposition term $\Lambda(r, t)$

- Birth/death term for population; deposition are due to ground launch, but there is also the option for dedicated "cleanup" events
- We assume it is separable,

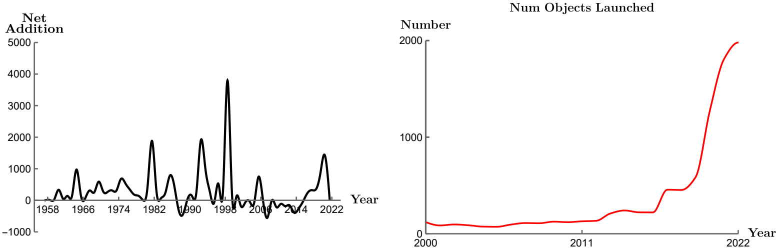
$$\Lambda(r, t) = R(r)T(t)$$

where certain orbital "bands" are targeted,

$$R(r) = \sum_{k=1}^{n} a_k \exp\left(-\frac{(r - r_k)^2}{\sigma_k^2}\right),$$

- $T(t)$ interpolated from historical launch numbers

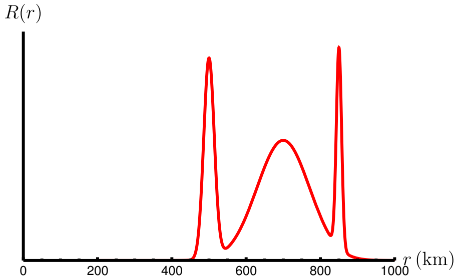
Temporal deposition levels



(Left) Net addition of objects to the LEO population since 1957, exhibiting a period of approximately $T = 5$ y. **(Right)** Number of satellites launched into LEO each year from 2000 to 2021, maintained by the UN Office for Outer Space Affairs; `unoosa.org`.
For most of the simulations we work with

$$T(t) = I_{2022}(1 + k_2 \sin(2\pi f(t - t_0)))$$

Deposition locations $R(r)$



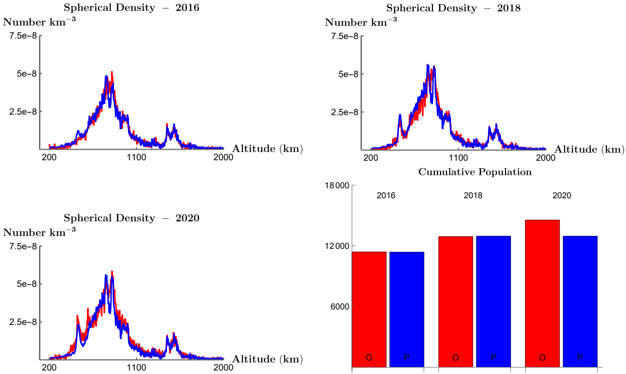
This is motivated by inspection of the space debris densities from 2014 through 2022.

Numerical simulations

- The numerical solutions of the model are obtained via the Crank-Nicolson method.
- First we propagate the 2014 population to 2020 to validate the model, then then we propagate the 2022 population into the future.
- Simulations take $\approx$ 2.5 minutes on a commercial laptop.

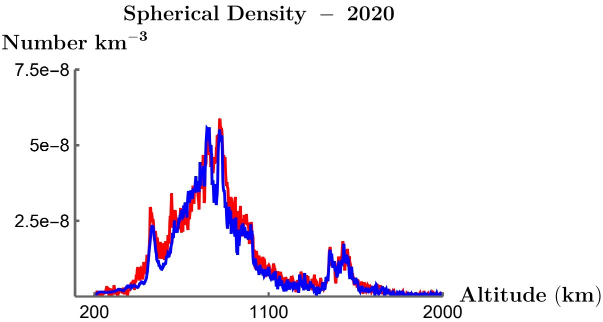
The `python` codes are available at
`github.com/jurkiew4/Space-Debris-Analytic-Model`

Simulations from the past



Comparison of observed (red) population and propagation of 2014 (blue) populations under the action of ground launches in 2016, 2018 and 2020 respectively, along with the population growth (lower right). Data source: Space-Track.org.

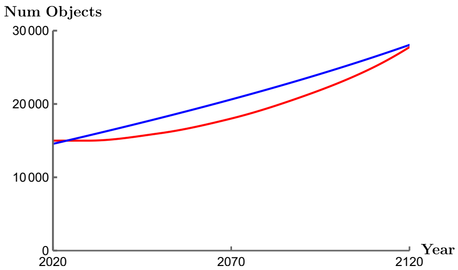
Simulations from the past



There is a good agreement in both the L^1 and L^∞ norms. The developing spike at 500 km coincides with a popular deposition altitude, including the Starlink satellite constellation.

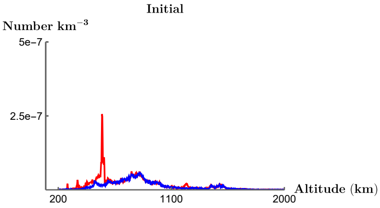
Comparison with LEGEND

LEGEND is NASA's primary prediction tool, but is not available to the public, only some of its predictions are.



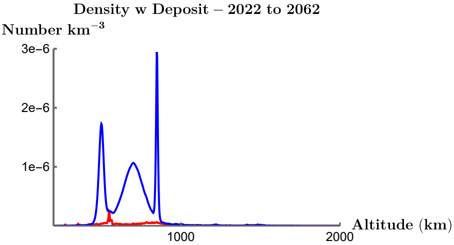
Comparison of total population forecasts for 2020 population by our model (blue) to the predictions of LEGEND (red) from 2020 to 2120. The LEGEND data are reproduced from figures presented in Liou (2011).

Simulations



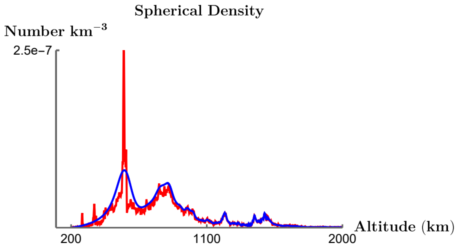
Initial densities (left) in 2020 (blue) and 2022 (red), along with densities propagated 100 years into the future in the event of *no further deposition* **(!)**. The substantial difference at the 500 km level is due to the Russian anti-satellite weapon test (the Kosmos-1408 destruction) in November 2021.

The "head-in-the-sand" attitude



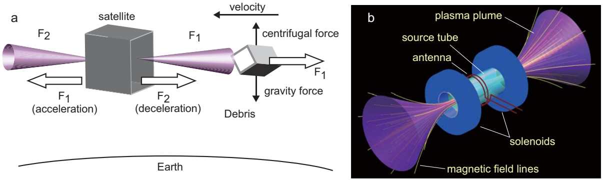
Evolution of the 2022 populations (initial in red, propagated in blue) to 2062 under continued deposition at 2022 rate. The growth in this scenario over 40 years is a full order of magnitude greater than the no-launch scenario after 100 years.

The importance of parameter choices

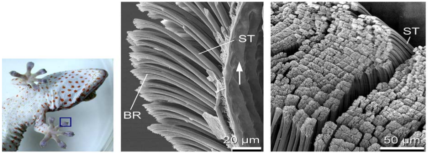


Propagation of the 2022 density (red) to 2072 with enlarged diffusivity parameters. Catastrophic growth is avoided.

Cleaning up in space

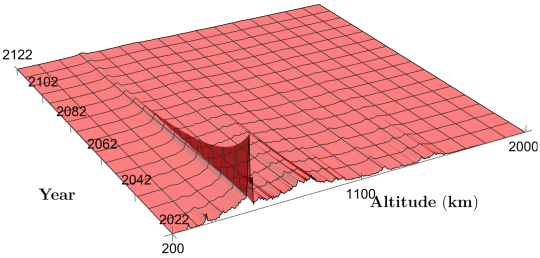


A bidirectional plasma thruster decelerates debris objects so that they burn up in the atmosphere (Takahashi *et al.*, Sci. Rep., 2018); laboratory demonstration on Earth. Other removal mechanisms include nets, harpoons and "gecko feet" (Enrico Stoll, TU Berlin).



Simulations with “negative birth” term

$\Lambda(r, t)$ can be used also as an active removal term.



Propagation of the 2022 population to 2072 under action of active removal of the form $\Lambda(r, t) = -\eta u(r, t)$. The removal amounts to 5% per year at all altitudes.

Summary

Our population dynamics model provides accurate approximation to growth of space debris population from the past to the present, allowing for predictions and policy considerations for the future.


EIN ALL FÜR ALLE

schätzen

2019 ahnten sie, dass Starlink nicht nur ein Segen ist. Denn kurz nachdem die ersten Exemplare im Orbit angekommen waren, zogen Perlenketten aus hellen Punkten über den Nachthimmel – Starlink-Satelliten, deren Solarmodule Sonnenlicht reflektieren. »Noch werden

Versicherung abschließen müssen, die für Schäden aller Art aufkommt. Nach derzeitiger Rechtslage zahlt in vielen europäischen Ländern, darunter Deutschland, der Steuerzahler.

Am Ende könnte man so Standards setzen, glaubt Nienaß, selbst für die großen Konzerne. In der Digitalwirtschaft sei das ja auch gelungen: Dort müssen sich Internetgiganten wie Google und Facebook mittlerweile am EU-Datenschutz orientieren, wenn sie Dienste in Europa anbieten

Die Zeit, July 21st, 2022

zeit.de/2022/30/raumfahrt-spacex-starlink-niklas-nienass-weltraumpolitik

Possible future research directions

- More precise parametrization is needed to better match available data. In particular, the diffusivity shows only a poorly defined pattern
- Hybrid discrete-continuous approach: modeling of large "rogue" objects, e.g. Envisat (26 m $\times$ 10 m $\times$ 5 m, 8.2 t, will linger around for another 150 years)
- Link up and compare with other models, e.g. ORDEM (NASA) and MASTER (ESA).
- Structured population dynamics: distinguish objects according to their momentum or "ballistic coefficient", mass/area M/A

Future research

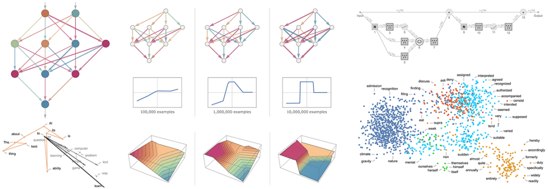
- Tailor the clean-up form of $\Lambda(r,t)$ to more specific active removal strategies
- Optimization problem: find $\Lambda(r,t)$ that minimizes some "cost/danger functional"

$$I(\Lambda) = \int_{t_0}^{t_1} \int_{r_E+200}^{r_E+2000} F(u,\Lambda) r^2 \,\mathrm{d}r\,\mathrm{d}t.$$

- Inform policy by minimizing collision risk by some metric

Ongoing "conversations"

Can Large Language Models (LLMs) be used as a tool to accelerate the pace of scientific research?



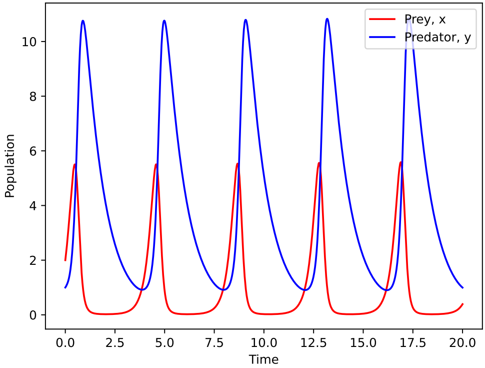
stephenwolfram.com/2023/02/what-is-chatgpt-doing-and-why-does-it-work/

Recent examples from chemistry and pure (!) mathematics

1. Autonomous chemical research with large language models, D. Boiko *et al.*, *Nature* **624**:570-578 (2023)
2. Mathematical discoveries from program search with large language models, B. Romera-Paredes *et al.*, *Nature* **625**:468-475 (2024)

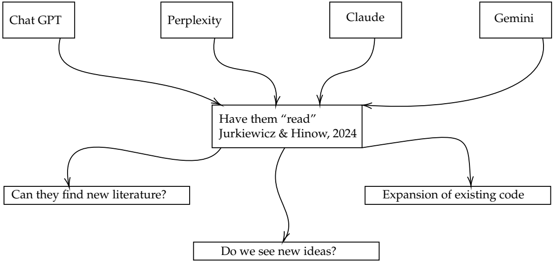
The "zeroth" example

Have ChatGPT with the GPT-3.5 LLM (which is already outdated) write a `python` code to solve the Lotka–Volterra equations.



Of course, not all output can be trusted 100 %.

Our approach



1. We ask the same set of questions to all LLMs.
2. Which LLMs are best suited for what purpose?
3. We attempt to quantify how interesting we find each conversation.

github.com/Florez001/GenAI_chats_2

Capabilities and costs of chatbots

	ChatGPT Plus	Gemini Advanced	Perplexity.AI	Claude 3 Opus
Can accept a .pdf	✓	×	✓	✓
Can export a conversation	✓	✓	✓	×
Number of queries	∞	∞	∞	200
Cost (USD/month)	20	20	20	20
Web Access	×	✓	✓	×

Overall, the premium LLMs are much better than the free versions.

Earliest novelties (emphasis mine)

1. GPT-4: `Accounting for orbital resonances that can lead to clustering of debris objects in certain regions, affecting collision risks and debris distribution.` + paper J. Rosengren, 2018
2. Gemini: `The current model assumes a spherically symmetric debris distribution. Introduce more detailed orbital mechanics to account for how specific orbits (inclination, eccentricity) influence debris concentration and collision likelihood.`
3. Claude: `Instead of using a continuous collision term that depends on the average debris density, you could model individual collision events as discrete, random occurrences. This could be done using a Poisson process ...`

Acknowledgments

- Kerri Cahoy (Massachusetts Institute of Technology), Colin McInnes (University of Glasgow), and Jer-Chyi Liou (NASA) for sharing literature and valuable comments
- the editors and reviewers of *CAMC*
- Support for Undergraduate Research Fellows (SURF) award from UWM to Victor Florez
- Andreas Deutsch (Dresden University of Technology) for his hospitality during my sabbatical in 2023

J. Jurkiewicz and P. Hinow, A Population Dynamics Approach to the Distribution of Space Debris in Low Earth Orbit, *Communications on Applied Mathematics and Computation* **6**:340-353 (2024)
arxiv.org/abs/2210.16179
github.com/jurkiew4/Space-Debris-Analytic-Model

Thank you for your attention