International Meetings on Differential Equations and Their Applications

Juan J. Nieto

Nonlinear fractional differential equations

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Nonlinear fractional differential equations

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Juan J. Nieto

Nonlinear fractional differential equations

Juan J. Nieto

We present some basic aspects of fractional calculus and fractional differential equations.

Some simple nonlinear fractional equations are considered. Some of them are easily solved, but others present some new difficulties and problems. As a model we focus on the nonlinear logistic equation.

References:

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- I. Area, J.J. Nieto, Power series solution of the fractional logistic equation. Physica A: Statistical Mechanics and its Applications 573 (2021), 125947.J.J. Nieto Solution of a fractional logistic ordinary differential equation. Applied Mathematics Letters 123 (2022), 107568.
- J.J. Nieto, Fractional Euler numbers and generalized proportional fractional logistic differential equation. Fractional Calculus and Applied Analysis 25 (2022), pp. 876-886.
- J.L. Wei, G.C. Wu, B.Q. Liu, J.J. Nieto, An optimal neural network design for fractional deep learning of logistic growth. Neural Computing and Applications, In Press, 2023. https://doi.org/10.1007/s00521-023-08268-8

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Fractional World: from calculus to nonlinear differential equations

- Fractional calculus
- Fractional Differential Equations
- Nonlinear Fractional Differential Equations
- Some models and applications

Fractional calculus is the study of integrals and derivatives of any order, not only integer. There are several definitions of fractional integral and fractional derivative due to Riemann, Liouville, Weyl, Hilbert, etc.

In 1695 Leibniz wrote a letter to L’Hôpital: *Can the meaning of derivatives with integer order be generalized to derivatives with non-integer orders?*

“WHAT IF $n = \frac{1}{2}$ IN $\frac{d^n f(x)}{dx^n}$”

It will lead to a paradox, from which one day useful consequences will be drawn

$$I^1 f(t) = \int_0^t f(s)ds$$

$$I^2 f(t) = I^1(I^1 f)(t) = \int_0^t (t-s)f(s)ds$$

$$I^n f(t) = \frac{1}{(n-1)!}\int_0^t (t-s)^{n-1} f(s)ds$$

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1} f(s)ds \qquad \alpha > 0, f \in L^1(0,T)$$

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} f(s) ds \qquad \alpha > 0, f \in L^1(0,T)$$

$$I^\alpha : L^1(0,T) \to L^1(0,T)$$

$$\alpha > 0, \beta > 0 : I^\alpha \circ I^\beta = I^{\alpha+\beta}$$

$$\alpha > 0, \lambda > -1 : I^\alpha t^\lambda = \frac{\Gamma(\lambda+1)}{\Gamma(\lambda+\alpha+1)} t^{\lambda+\alpha}$$

$$I^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} f(s) ds$$

k(t,s)

The Prabhakar fractional integral with base point 0 is defined by

$$\mathbb{P}^{\gamma}_{\alpha,\beta,\lambda}\sigma(t) = \int_0^t e^{\gamma}_{\alpha,\beta}(\lambda; t-s)\sigma(s)ds.$$

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} f(s)ds$$

k(t,s)

The Prabhakar fractional integral with base point 0 is defined by

$$\mathbb{P}^{\gamma}_{\alpha,\beta,\lambda}\sigma(t) = \int_0^t e^{\gamma}_{\alpha,\beta}(\lambda; t-s)\sigma(s)ds.$$

$$E^{\gamma}_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_n}{\Gamma(n\alpha+\beta)} \frac{z^n}{n!} \qquad e^{\gamma}_{\alpha,\beta}(\lambda; t) = t^{\beta-1} E^{\gamma}_{\alpha,\beta}(\lambda t^{\alpha}),$$

$$\mathbb{P}^{\gamma}_{\alpha,\beta,0}\sigma(t) = \int_0^t e^{\gamma}_{\alpha,\beta}(0, t-s)\sigma(s)ds = \frac{1}{\Gamma(\beta)} \int_0^t (t-s)^{\beta-1}\sigma(s)ds = I^{\beta}\sigma(t).$$

$$\mathbb{D}^{\gamma}_{\alpha,\beta,\lambda}\sigma(t) = \frac{d}{dt}\mathbb{P}^{-\gamma}_{\alpha,1-\beta,\lambda}\sigma(t)$$

$$\mathbb{D}^{\gamma}_{\alpha,\beta,\lambda}\sigma(t) = \mathbb{P}^{-\gamma}_{\alpha,1-\beta,\lambda}\sigma'(t)$$

$$D^1(t^n) = nt^{n-1}$$

$$D^2(t^n) = n(n-1)t^{n-2} = \frac{\Gamma(n+1)}{\Gamma(n-2+1)}t^{n-2}$$

$$D^\alpha(t^n) = \frac{\Gamma(n+1)}{\Gamma(n-\alpha+1)}t^{n-\alpha}$$

$$D^{1/2}t^1 = \frac{\Gamma(2)}{\Gamma(\frac{1}{2}+1)}t^{1/2} = \frac{2}{\pi}\sqrt{t}$$

$$D^{1/2}1 = \frac{1}{\sqrt{\pi}}t^{-1/2} \neq 0$$

$$D^{1/2}t^{1/2} = \Gamma(\frac{1}{2}+1)t^0 = \Gamma(3/2)$$

$$0 < \alpha < 1 : D^\alpha f = D^1 I^{1-\alpha} f$$

$$D^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t (t-s)^{-\alpha} f(s) ds$$

Riemann-Liouville fractional derivative

$$D^\alpha 1 \neq 0$$

$$D^\alpha t^\lambda = \frac{\Gamma(\lambda+1)}{\Gamma(\lambda-\alpha+1)} t^{\lambda-\alpha}$$

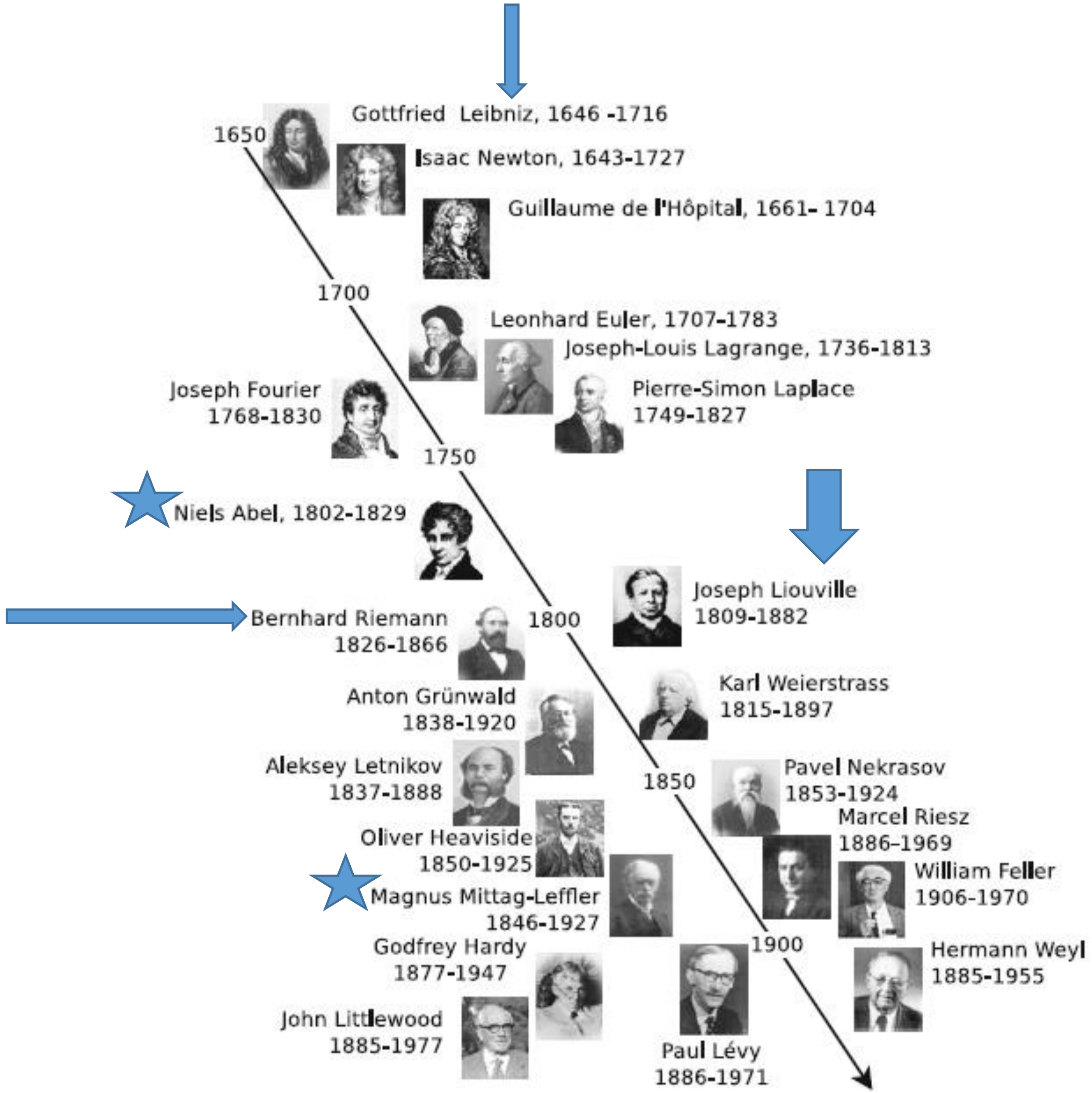
$$D^\alpha c = \frac{c}{\Gamma(1-\alpha)} t^{-\alpha}$$

$$D^\alpha f = I^{1-\alpha} D^1 f$$

$${}^C D^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} f'(s) ds$$

Liouville-Caputo fractional derivative

$${}^C D^\alpha 1 = 0$$


Gottfried Leibniz, 1646 -1716
Isaac Newton, 1643-1727
Guillaume de l'Hôpital, 1661- 1704
1650
1700
Leonhard Euler, 1707-1783
Joseph-Louis Lagrange, 1736-1813
Pierre-Simon Laplace 1749-1827
Joseph Fourier 1768-1830
1750
Niels Abel, 1802-1829
Joseph Liouville 1809-1882
Bernhard Riemann 1826-1866
1800
Karl Weierstrass 1815-1897
Anton Grünwald 1838-1920
Aleksey Letnikov 1837-1888
1850
Pavel Nekrasov 1853-1924
Marcel Riesz 1886–1969
Oliver Heaviside 1850-1925
William Feller 1906-1970
Magnus Mittag-Leffler 1846-1927
Godfrey Hardy 1877-1947
1900
Hermann Weyl 1885-1955
John Littlewood 1885-1977
Paul Lévy 1886-1971

$u' > 0 \rightarrow$ u increasing

$D^{\alpha}u > 0 \rightarrow$ u increasing?

Mean Value Theorem

Many Fractional Mean Value Theorem?

Fractional Differential Equations

$$D^{\alpha}u = 0$$

$$D^{\alpha}u = D^{1}I^{1-\alpha}u = 0 \Rightarrow I^{1-\alpha}u = c$$

$$I^{\alpha}I^{1-\alpha}u = I^{\alpha}c = \frac{c}{\Gamma(\alpha+1)}t^{\alpha}$$

$$I^{1}u = \frac{c}{\Gamma(\alpha+1)}t^{\alpha}$$

$$D^{1}I^{1}u = u = \frac{c}{\Gamma(\alpha+1)}\alpha t^{\alpha-1} = \frac{c}{\Gamma(\alpha)}t^{\alpha-1}$$

$$D^{\alpha}u = 0 \Leftrightarrow ct^{\alpha-1}, c \in R$$

$$D^\alpha u = 0$$

$$D^\alpha u = D^1 I^{1-\alpha} u = 0 \Rightarrow I^{1-\alpha} u = c$$

$$I^\alpha I^{1-\alpha} u = I^\alpha c = \frac{c}{\Gamma(\alpha+1)} t^\alpha$$

$$I^1 u = \frac{c}{\Gamma(\alpha+1)} t^\alpha$$

$$D^1 I^1 u = u = \frac{c}{\Gamma(\alpha+1)} \alpha t^{\alpha-1} = \frac{c}{\Gamma(\alpha)} t^{\alpha-1}$$

$$D^\alpha u = 0 \Leftrightarrow c t^{\alpha-1}, c \in R$$

$$D^\alpha u = f \Leftrightarrow u(t) = I^\alpha f(t) + c t^{\alpha-1}$$

$$0 < \alpha < 1$$

$$D^\alpha u = \lambda u, \lambda \neq 0$$

$$u(t) = c\,\Gamma(\alpha)\,t^{\alpha-1}\,E_{\alpha,\alpha}(\lambda t^\alpha)$$

Mittag-Leffler function

$$D^\alpha u = \lambda u + f, \lambda \neq 0$$

$$u(t) = c\,\Gamma(\alpha)\,t^{\alpha-1}\,E_{\alpha,\alpha}(\lambda t^\alpha) + \int_0^t (t-s)^{\alpha-1} E_{\alpha,\alpha}(\lambda (t-s)^\alpha) f(s) ds$$

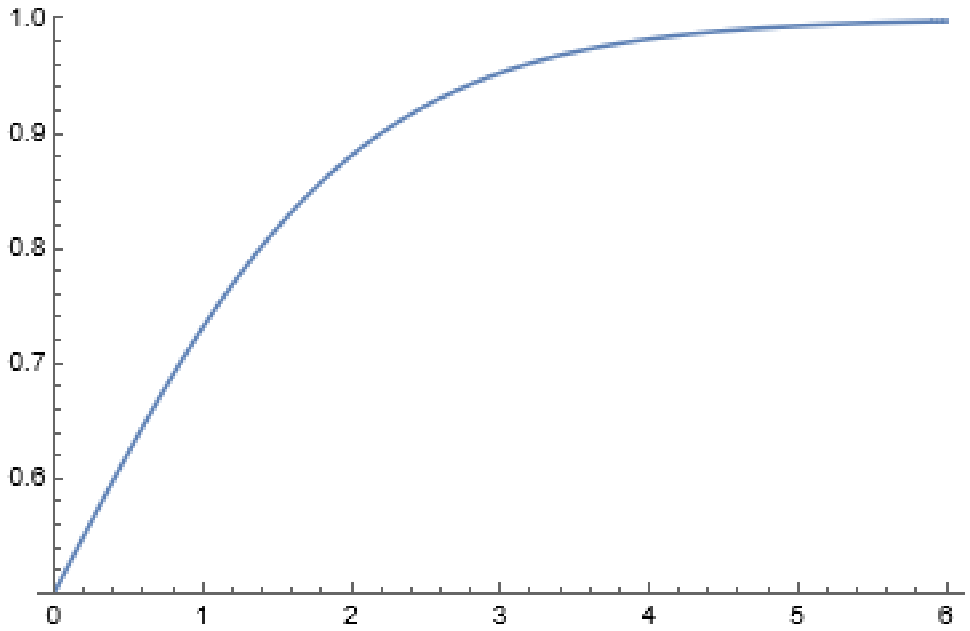
$$\lim_{t \to 0^+} t^{1-\alpha} u(t) = c$$

Nonlinear Fractional Differential Equations

$$u'(t) = ku(t)(1 - u(t)), \quad t \geq 0$$

$$u(t) = \frac{u_0}{u_0 + (1 - u_0)\exp(-kt)}, \quad t \geq 0$$

Fractional logistic ODE



Physica A 573 (2021) 125947

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Power series solution of the fractional logistic equation

I. Area [a,1,*], J.J. Nieto [b,1]

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[b] Instituto de Matemáticas, Universidade de Santiago de Compostela, 15782 Santiago de Compostela, Spain

$$D^{\alpha}v = v(1-v), \quad 0 < \alpha \leq 1,$$

$$v(t) = \sum_{n=0}^{\infty} b_n(\alpha)(t^{\alpha})^n.$$

$$b_{n+1}(\alpha) = \frac{\Gamma(n\alpha+1)}{\Gamma((n+1)\alpha+1)}\left[b_n(\alpha) - \sum_{j=0}^{n} b_j(\alpha)b_{n-j}(\alpha)\right], \quad n \geq 0,$$

$$b_0(\alpha) = v(0)$$

$$\mathcal{D}^{\alpha} x(t) = x(t) \cdot [1 - x(t)]$$

$$\frac{x(t) - x^2(t)}{(1 - x(t))^{2/\alpha}} = \frac{x_0 - x_0^2}{(1 - x_0)^{2/\alpha}} \cdot e^t.$$

$$\frac{x(t)-x^2(t)}{(1-x(t))^{2/\alpha}}=\frac{x_0-x_0^2}{(1-x_0)^{2/\alpha}}\cdot e^t.$$

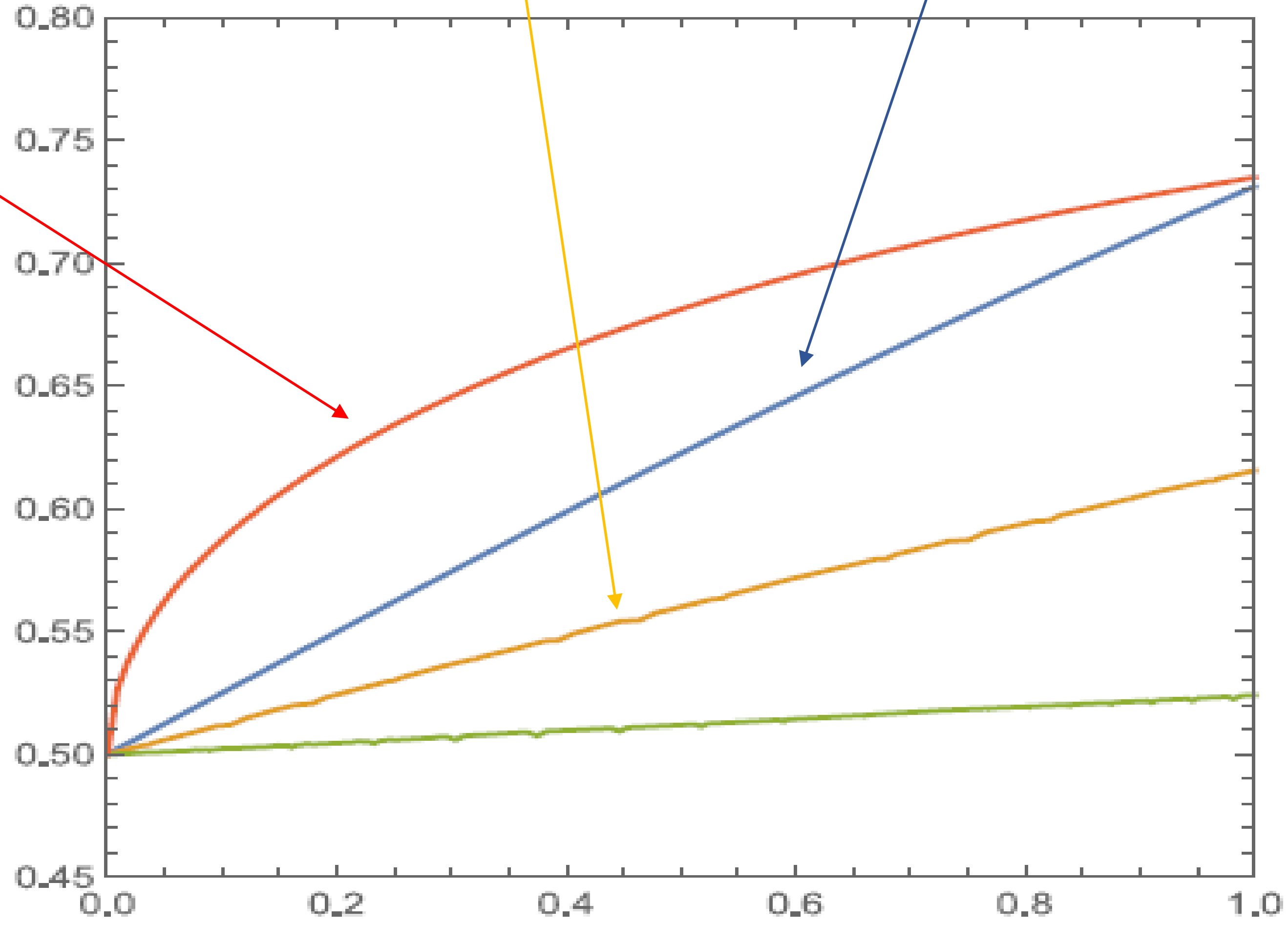
Caputo-Fabrizio
fractional DE $\alpha = 1/2$

Classical Logistic ODE
$\alpha = 1$

Caputo fractional
DE $\alpha = 1/2$

x(0)=1/2

Caputo-Fabrizio
fractional DE $\alpha = 0.1$



Applied Mathematics Letters 123 (2022) 107568

Contents lists available at ScienceDirect

Applied Mathematics Letters

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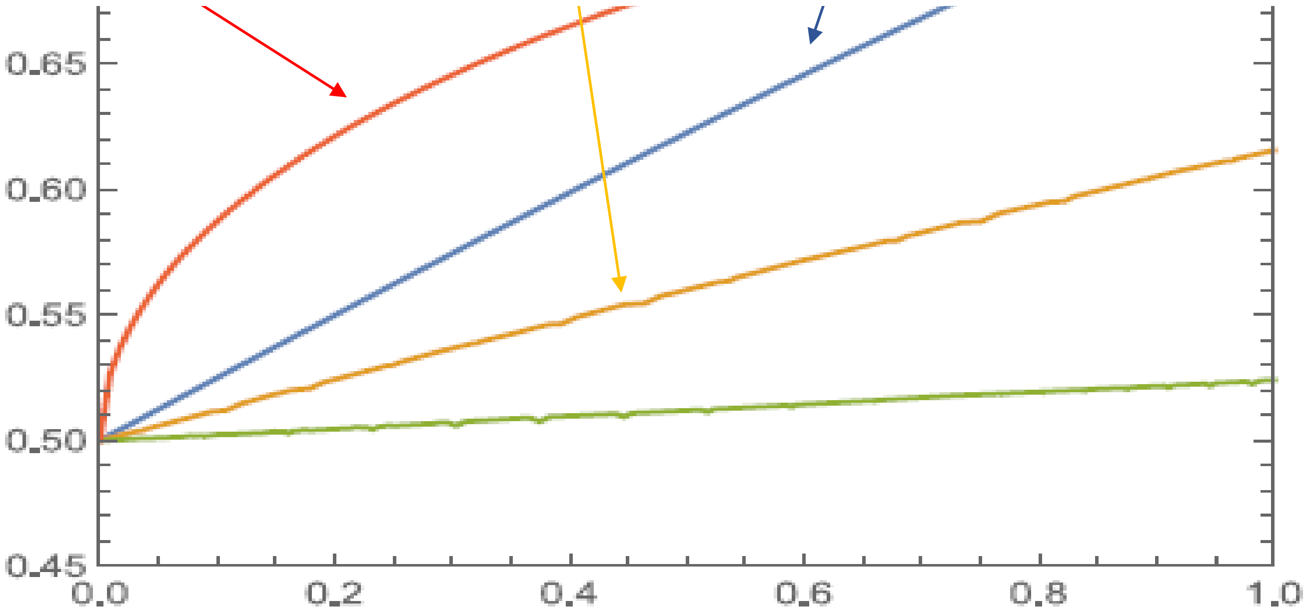
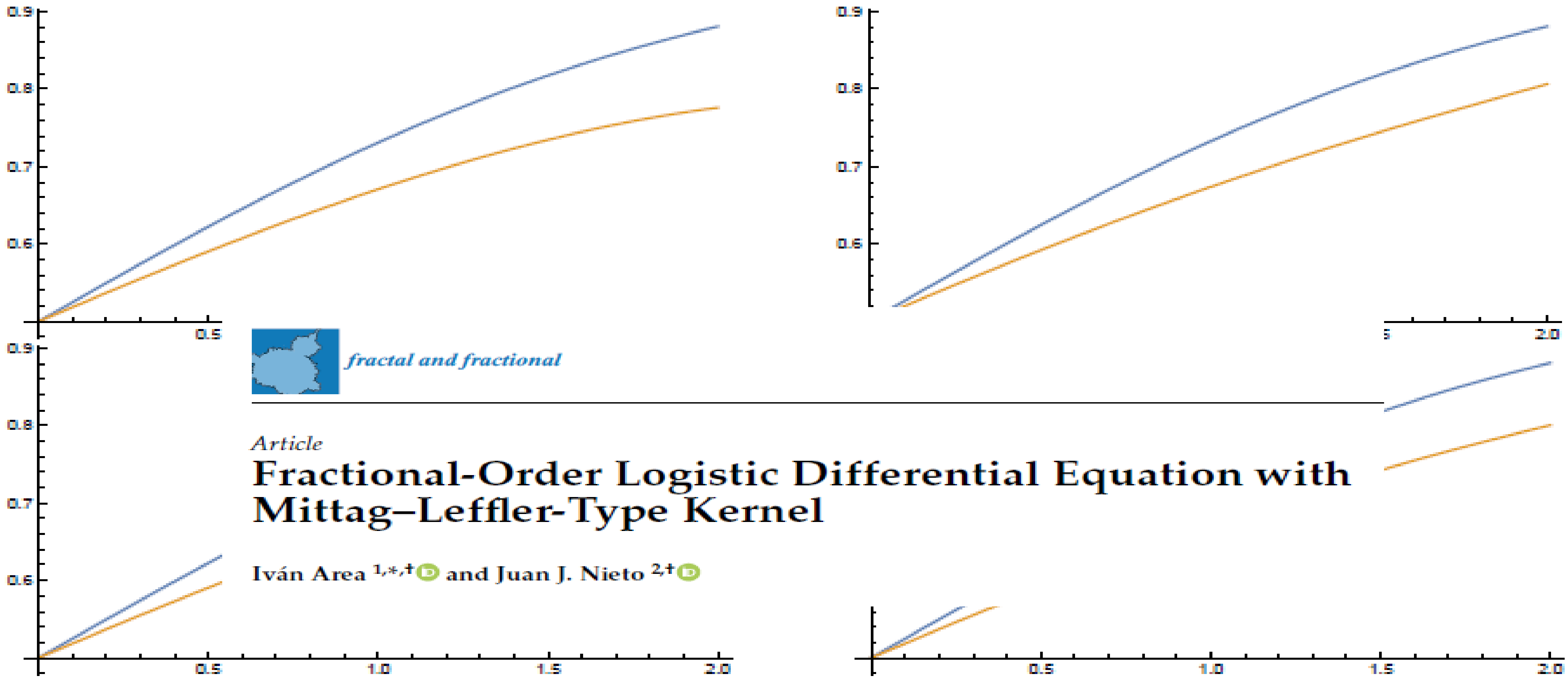
Solution of a fractional logistic ordinary differential equation

Juan J. Nieto

Caputo-Fabrizio fractional DE α = 0.1





fractal and fractional

Article

Fractional-Order Logistic Differential Equation with Mittag–Leffler-Type Kernel

Iván Area [1,*,†] and Juan J. Nieto [2,†]

Logistic function solution with $x(0) = 1/2$, in blue, as well as some approximations of the solution to the Caputo–Fabrizio logistic differential Equation (29) in $[0, 2]$ for $\alpha = 0.75$, in orange. From left to right and top to bottom the approximations are shown for $n = 3$, $n = 5$, $n = 7$,

Applications

Differential Equations and Dynamical Systems
https://doi.org/10.1007/s12591-022-00622-x

ORIGINAL RESEARCH

Application of Non-singular Kernel in a Tumor Model with Strong Allee Effect

Subhas Khajanchi[1] · Mrinmoy Sardar[2] · Juan J. Nieto[3]

$$\frac{dT}{dt} = \alpha T\left(1 - \frac{T}{k}\right)\left(T - c\right),$$

Classical Liouville-Caputo fractional derivative

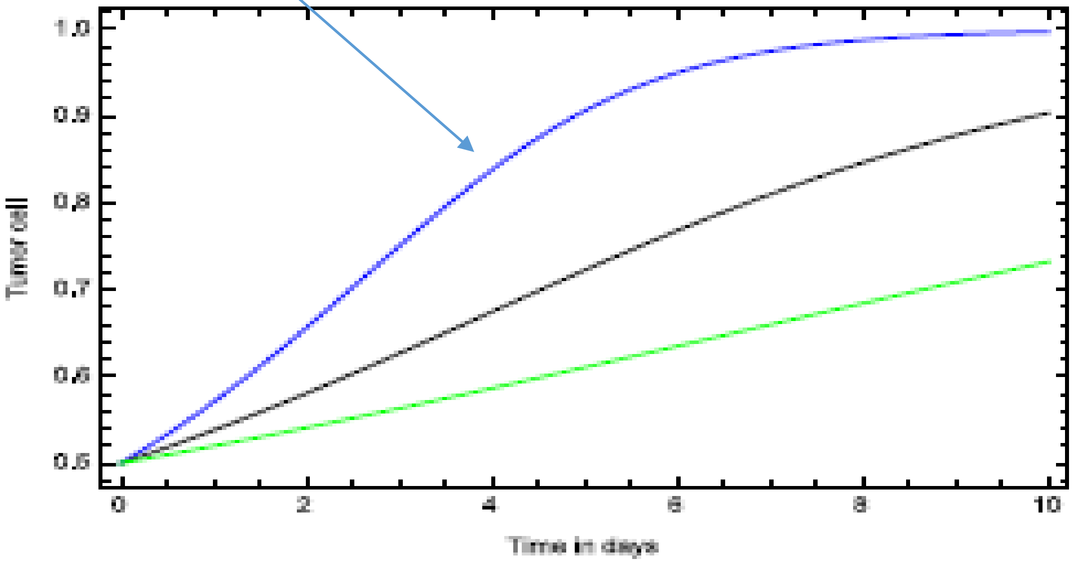


Fig. 1 Solution of the ordinary logistic differential equation with strong Allee effect for the initial value $T_0 = 0.5$. Classical logistic equation with strong Allee effect (blue colour), fractional Caputo-Fabrizio logistic equation with strong Allee effect (7) for $\lambda = 1/2$ (black) and for $\lambda = 1/4$ (green)

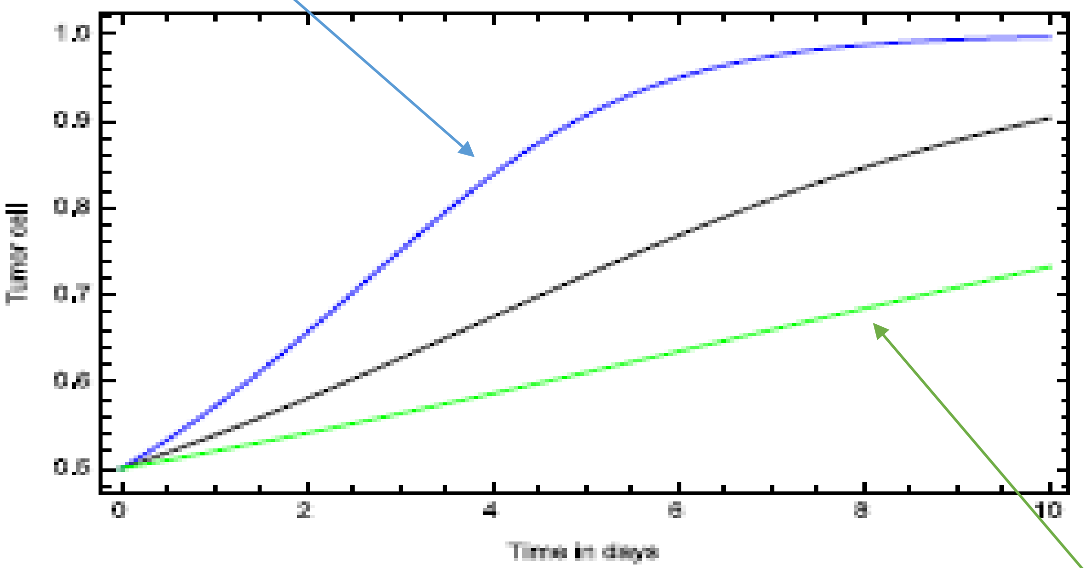
$$e^{lt} \cdot (T_2(0))^{\frac{1}{3}} T_0^{\left(\frac{a_3}{a_2} - \frac{1}{3}\right)} (T_3(0))^{\left(\frac{1}{6} - \frac{a_3}{2a_2}\right)} \left(\frac{T_0 - c}{T_0 - k}\right)^{\frac{3a_1a_3 - a_1a_2}{6a_2(c-k)}} = (T_2(t))^{\frac{1}{3}} (T(t))^{\left(\frac{a_3}{a_2} - \frac{1}{3}\right)}.$$

$$(T_3(t))^{\left(\frac{1}{6} - \frac{a_3}{2a_2}\right)} \left(\frac{T(t) - c}{T(t) - k}\right)^{\frac{3a_1a_3 - a_1a_2}{6a_2(c-k)}}.$$

$$\Rightarrow \exp(lt) = \left(\frac{T_2(t)}{T_2(0)}\right)^{\frac{1}{3}} \left(\frac{T(t)}{T_0}\right)^{\left(\frac{a_3}{a_2} - \frac{1}{3}\right)} \left(\frac{T_3(t)}{T_3(0)}\right)^{\left(\frac{1}{6} - \frac{a_3}{2a_2}\right)} \left(\frac{(T(t) - c)(T_0 - k)}{(T(t) - k)(T_0 - c)}\right)^{\frac{3a_1a_3 - a_1a_2}{6a_2(c-k)}}.$$

Classical Liouville-Caputo fractional derivative

Fig. 1 Solution of the ordinary logistic differential equation with strong Allee effect for the initial value $T_0 = 0.5$. Classical logistic equation with strong Allee effect (blue colour), fractional Caputo-Fabrizio logistic equation with strong Allee effect (7) for $\lambda = 1/2$ (black) and for $\lambda = 1/4$ (green)



Caputo-Fabrizio derivative

$$e^{k} \cdot (T_2(0))^{\frac{1}{3}} T_0^{\left(\frac{a_3}{a_2}-\frac{1}{3}\right)} (T_3(0))^{\left(\frac{1}{6}-\frac{a_3}{2a_2}\right)} \left(\frac{T_0-c}{T_0-k}\right)^{\frac{3a_1a_3-a_1a_2}{6a_2(c-k)}} = (T_2(t))^{\frac{1}{3}} (T(t))^{\left(\frac{a_3}{a_2}-\frac{1}{3}\right)} .$$

$$(T_3(t))^{\left(\frac{1}{6}-\frac{a_3}{2a_2}\right)} \left(\frac{T(t)-c}{T(t)-k}\right)^{\frac{3a_1a_3-a_1a_2}{6a_2(c-k)}} .$$

$$\Rightarrow \exp(lt) = \left(\frac{T_2(t)}{T_2(0)}\right)^{\frac{1}{3}} \left(\frac{T(t)}{T_0}\right)^{\left(\frac{a_3}{a_2}-\frac{1}{3}\right)} \left(\frac{T_3(t)}{T_3(0)}\right)^{\left(\frac{1}{6}-\frac{a_3}{2a_2}\right)} \left(\frac{(T(t)-c)(T_0-k)}{(T(t)-k)(T_0-c)}\right)^{\frac{3a_1a_3-a_1a_2}{6a_2(c-k)}} .$$

Chaos, Solitons and Fractals 144 (2021) 110652

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Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Fondo COVID-19

Instituto de Salud Carlos III

Fractional model of COVID-19 applied to Galicia, Spain and Portugal

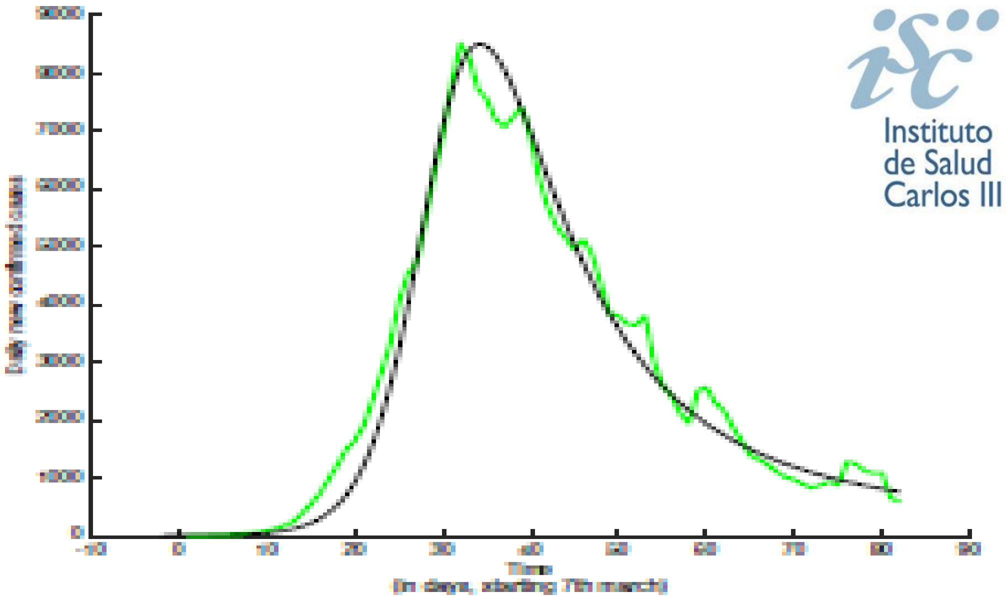
Faïçal Ndaïrou[a,b], Iván Area[b], Juan J. Nieto[c], Cristiana J. Silva[a], Delfim F.M. Torres[a,*]

[a] Center for Research and Development in Mathematics and Applications (CIDMA), Department of Mathematics, University of Aveiro, 3810-193 Aveiro, Portugal

[b] Universidade de Vigo, Departamento de Matemática Aplicada II, E. E. Aeronáutica e do Espazo, Campus de Ourense, 32004 Ourense, Spain

[c] Instituto de Matemáticas, Universidade de Santiago de Compostela, Santiago de Compostela 15782, Spain

$$
\begin{cases}
{}^{C}D^{\alpha}S(t) = -\beta\frac{I}{N}S - l\beta\frac{H}{N}S - \beta'\frac{P}{N}S, \\
{}^{C}D^{\alpha}E(t) = \beta\frac{I}{N}S + l\beta\frac{H}{N}S + \beta'\frac{P}{N}S - \kappa E, \\
{}^{C}D^{\alpha}I(t) = \kappa\rho_1 E - (\gamma_a + \gamma_i)I - \delta_i I, \\
{}^{C}D^{\alpha}P(t) = \kappa\rho_2 E - (\gamma_a + \gamma_i)P - \delta_p P, \\
{}^{C}D^{\alpha}A(t) = \kappa(1 - \rho_1 - \rho_2)E, \\
{}^{C}D^{\alpha}H(t) = \gamma_a(I + P) - \gamma_r H - \delta_h H, \\
{}^{C}D^{\alpha}R(t) = \gamma_i(I + P) + \gamma_r H, \\
{}^{C}D^{\alpha}F(t) = \delta_i I(t) + \delta_p P(t) + \delta_h H(t),
\end{cases}
$$



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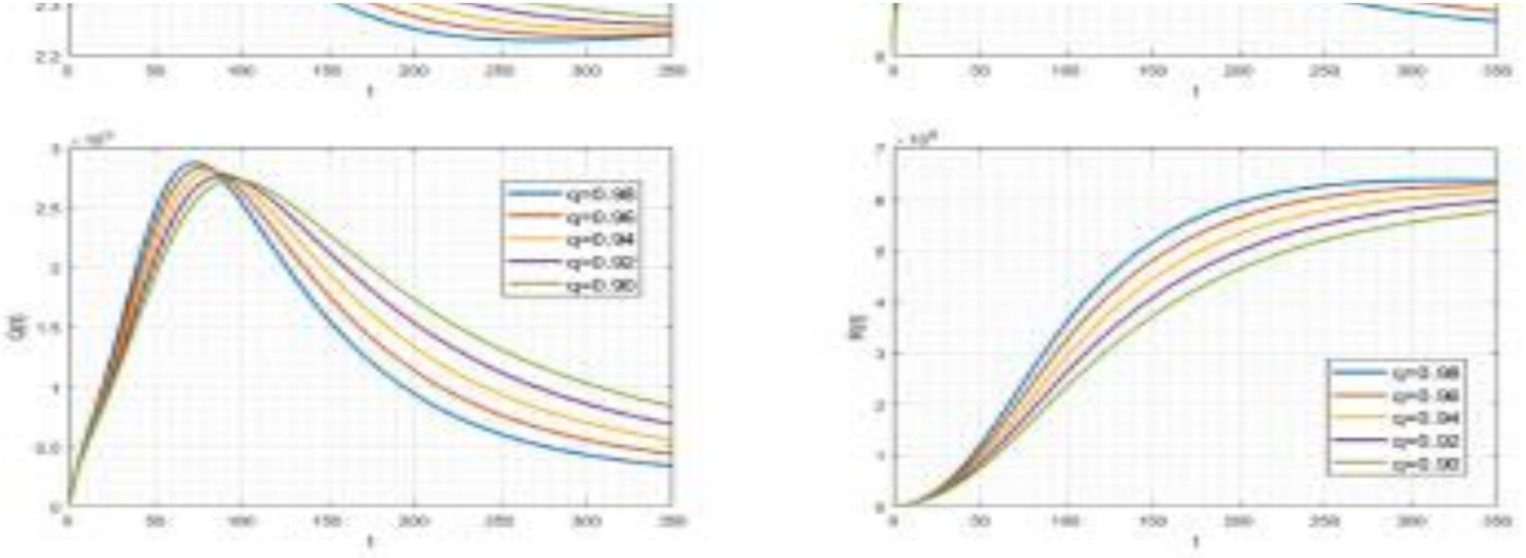
On a new and generalized fractional model for a real cholera outbreak

$$\begin{cases} \lambda^{q-1}\,{}_0^C\mathscr{D}_t^q S(t) = \Lambda - (\psi + \mu)S \\ \lambda^{q-1}\,{}_0^C\mathscr{D}_t^q I(t) = -(\alpha_1 + \mu + \gamma) \\ \lambda^{q-1}\,{}_0^C\mathscr{D}_t^q Q(t) = -(\alpha_2 + \mu + \epsilon \\ \lambda^{q-1}\,{}_0^C\mathscr{D}_t^q R(t) = -(\mu + \varphi_1)R(\\ \lambda^{q-1}\,{}_0^C\mathscr{D}_t^q V(t) = -(\varphi_2 + \mu)V(\\ \lambda^{q-1}\,{}_0^C\mathscr{D}_t^q C(t) = -\sigma C(t) + \theta I(\end{cases}$$



International Journal of Biomathematics
2250064 (20 pages)

DOI: 10.1142/S1793524522500644

World Scientific
www.worldscientific.com

Mathematical analysis of Hepatitis C Virus infection model in the framework of non-local and non-singular kernel fractional derivative

$$
\begin{aligned}
D^{\gamma}x(t) &= r\left(1-\frac{x+y}{k}\right)x-\beta_1 xy,\\
D^{\gamma}y(t) &= \beta_1 xy-\xi x-\beta_2 xv,\\
D^{\gamma}z(t) &= \Upsilon-\delta_1 z-\alpha zy,\\
D^{\gamma}w(t) &= \alpha yz-\delta_2 w,\\
D^{\gamma}v(t) &= \zeta wv-\beta_3 yv-\sigma v,\\
\gamma &\in [0,1]; \quad x(0),y(0),z(0),w(0),v(0)\geq 0.
\end{aligned}
$$

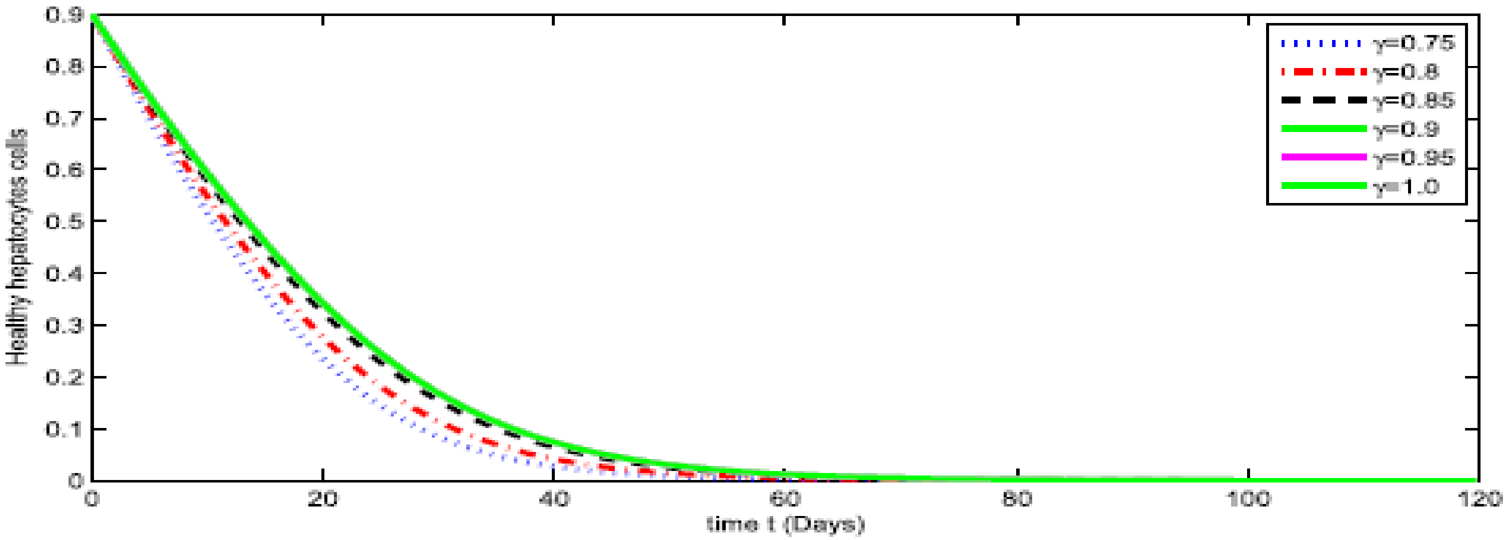


Fig. 1. Graph of approximate solution for healthy hepatocyte cells at different fractional values of γ.

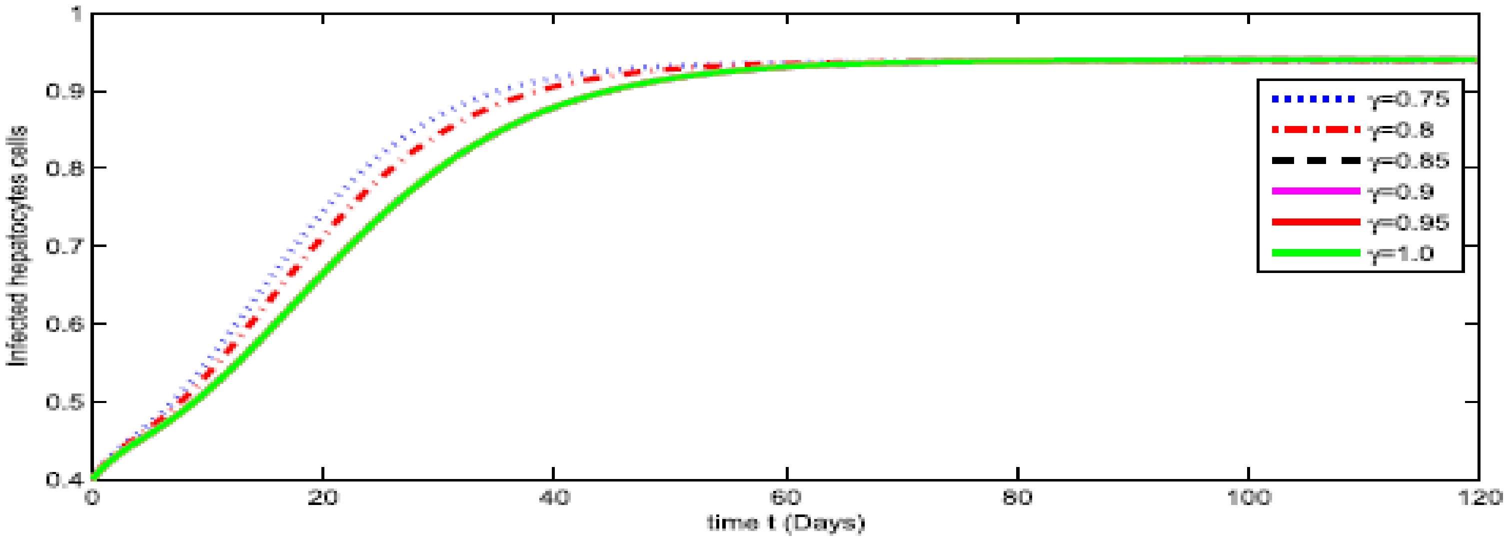


Fig. 2. Graph of approximate solution for infected hepatocyte cells at different fractional values of γ.

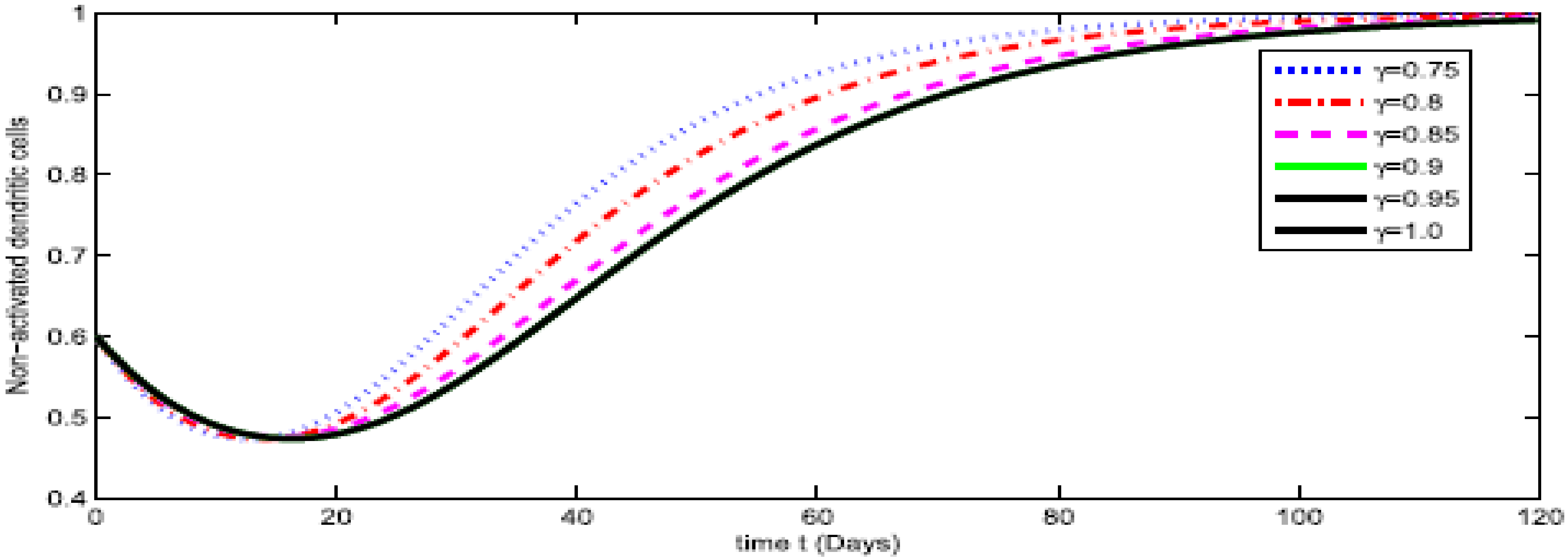
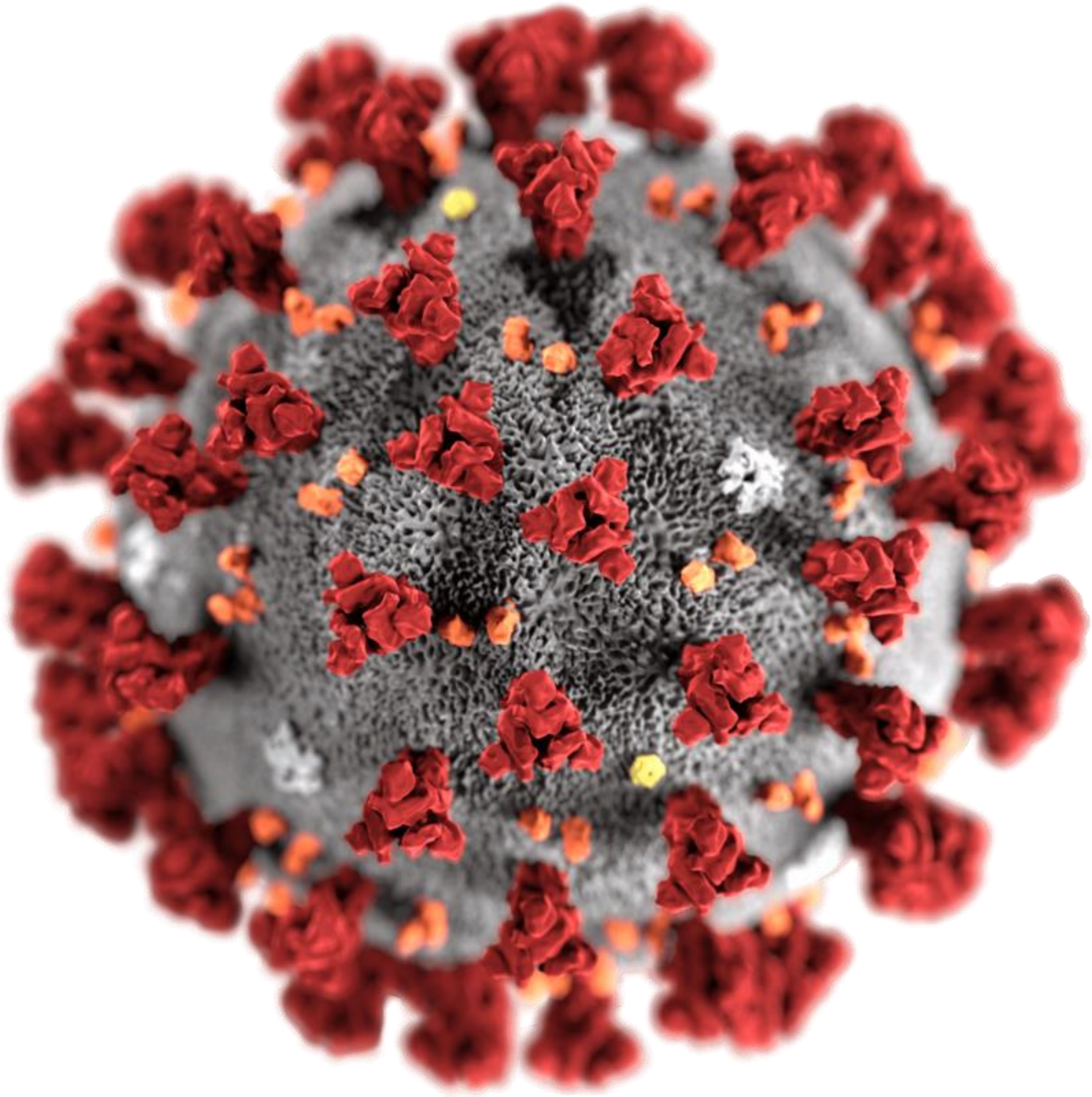
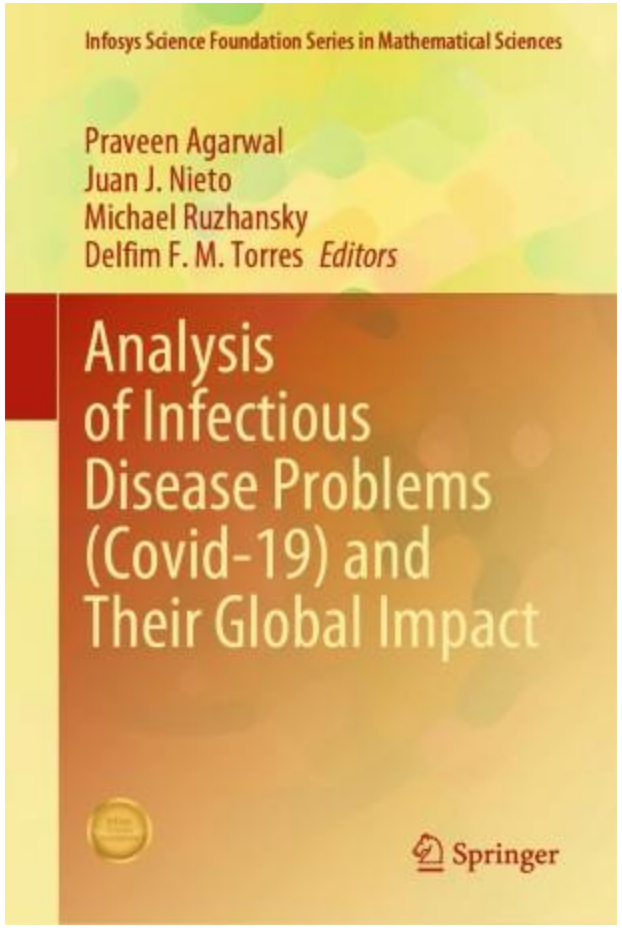


Fig. 3. Graph of approximate solution for non-activated dendritic cells at different fractional values of γ.





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Future directions

Neural Computing and Applications (2023) 35:10837–10846
https://doi.org/10.1007/s00521-023-08268-8

ORIGINAL ARTICLE

An optimal neural network design for fractional deep learning of logistic growth

Jia-Li Wei[1,2] · Guo-Cheng Wu[2] · Bao-Qing Liu[1] · Juan J. Nieto[3]

Received: 4 July 2022 / Accepted: 6 January 2023 / Published online: 9 February 2023

Abstract
This paper suggests a multi-layer neural network for deep learning based on fractional differential equations, and parallel computing is used to search an optimal structure. First, the Caputo derivative is approximated by L_1 numerical scheme and an unconstrained discretization minimization problem is presented. Then, parameters are adjusted by use of the Adam algorithm. Analytical approximate solutions of two fractional logistic equations (FLEs) are obtained which demonstrate the method's efficiency. Furthermore, with real-life data, fractional order and other parameters of FLEs are estimated by the gradient descent algorithm meanwhile. The proposed optimal NN method is used in forecasting. Through the comparative study, FLEs have more parameter freedom degrees and perform better than the classical logistic model.

Mathematical Methods in the Applied Sciences

A Digital Twin of a Compartmental Epidemiological Model based on a Stieltjes Differential Equation

Iván Area[a], F.J. Fernández[b], Juan J. Nieto[b,*], F. Adrián F. Tojo[b]

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Abstract

We introduce a digital twin of the classical compartmental SIR (Susceptible, Infected, Recovered) epidemic model and study the interrelation between the digital twin and the system. In doing so, we use Stieltjes derivatives to feed the data from the real system to the virtual model which, in return, improves it in real time. As a byproduct of the model, we present a precise mathematical definition of solution to the problem. We also analyze the existence and uniqueness of solutions, introduce the concept of Main Digital Twin and present some numerical simulations with real data of the COVID-19 epidemic, showing the accuracy of the proposed ideas.

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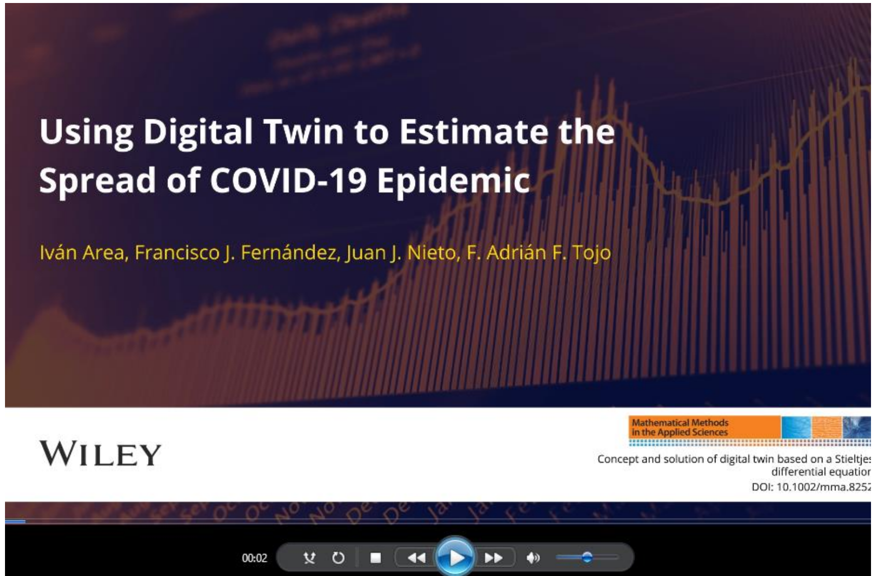
Digital Twin
for epidemic of COVID-19

Video at

https://onlinelibrary.wiley.com/doi/10.1002/mma.8252

https://twitter.com/CITMAga/status/1511267398405570562



Open Problems

Control of fractional systems

Sobolev spaces of fractional order

Fractional logistic equation

Fractional Laplacian

General M-L functions

Fractional models

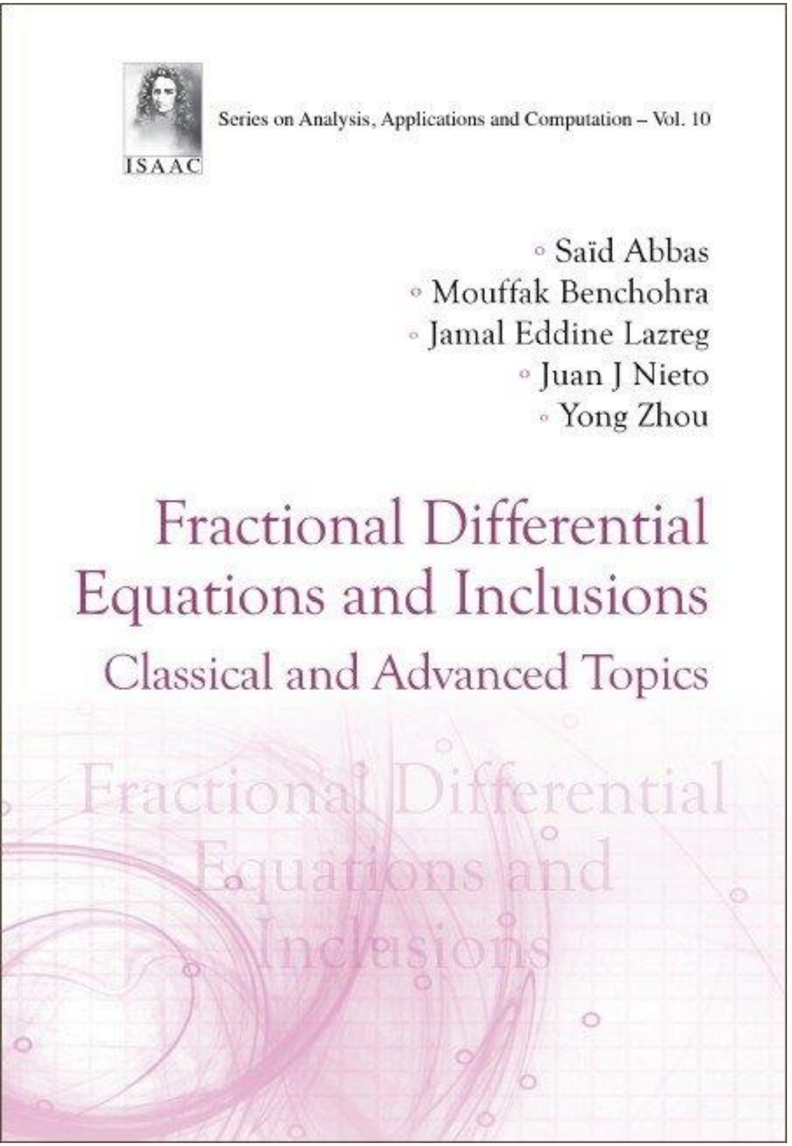
Fractional Lax-Milgram Theorem

Fractional Navier-Stokes equations

Fractional nonlinear models

Digital Twins

COVID-19 and future epidemics



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Conclusion: We have many new theoretical and applied problems to explore

Thank you for your attention

IMDETA

Nonlinear fractional differential equations

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