Long time behavior in a flow structure interaction

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International Meetings on Differential Equations and their Applications.
IMDETA -Winter- 2021, Lodz.
October 6, 2021

Acknowledgments

THANKS:

- National Science Foundation, NSF-DMS
- AFOSR -Office of Scientific Research

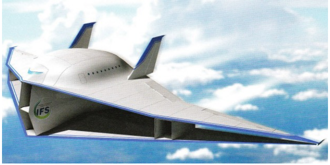
COLLABORATORS:

- Denise Bonheur, Universite de Bruxelles. Belgium.
- Igor Chueshov, Univ of Kharkov
- Earl Dowell, Duke University. USA
- Filippo Gazzola, University of Milano, Italy.
- Jason Howell, Carnegie Mellon, Pittsburgh, USA
- Roberto Triggiani, Univ of Memphis and Univ. of Virginia,
- Justin Webster, Univ of Maryland UMBC, USA

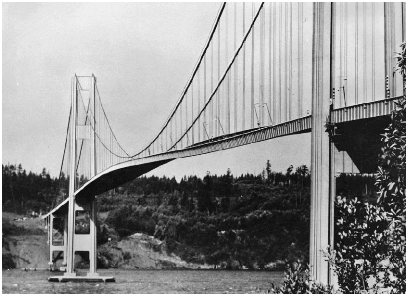
Flow Structure Interactions- Applications

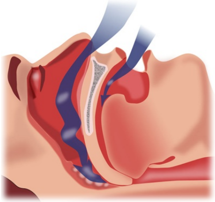
- **Aerodynamics.** Control of **flutter**.
  Flutter -sustained oscillations.
  - Subsonic, supersonic and transonic regimes;
- **Large space structures.** Large and thin. Highly oscillatory.
- **Medical Sciences**
  - Treatment of apnea
- **Engineering**
  - Oscillating Bridges and Buildings
  - Harvesting of energy-Windmills. Post flutter analysis.

flow-structure interaction, supersonic



flow-structure interaction, subsonic



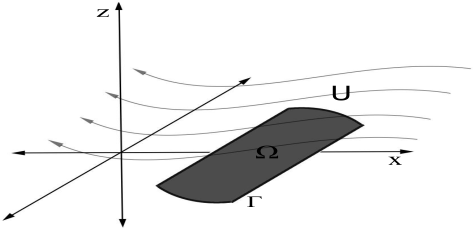


Outline

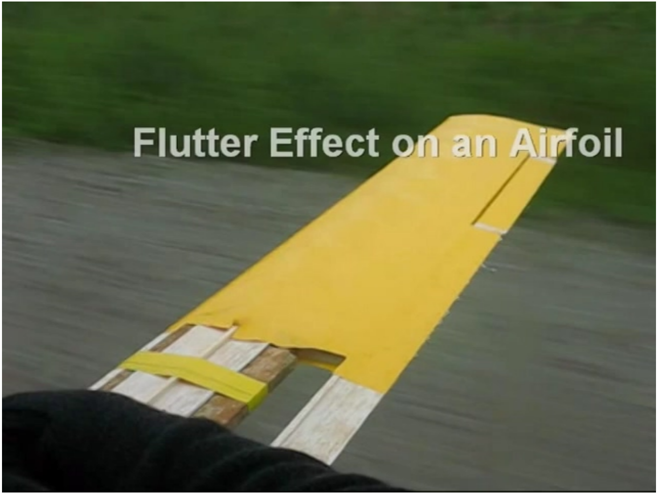
- **PDE Models**- Nonlinear Dynamics **Hyperbolic /Hyperbolic-like** with an **interface.**
- **Role of the Modeling** aligned with Experiments and Numerics.
- **Main Results**
  - **Representation as a wellposed Dynamical System** (S_t, X)**.**
  - **Stability and long time behavior**
    - **Global attractors for the structure only.**
    - **Control of the flutter -strong stability to an equilibria for the full interaction.**
  - Ideas about the Proofs - dynamical system theory.
  - Conclusions and **Open Problems**

The Model.

- Thin, flexible plate, **moving with a velocity U.**
  $M = \frac{velocity}{local\ speed\ of\ sound}$ ($U = 1$ normalised speed of sound).
- Ω is a closed, two-dimensional domain (smooth) in the x-y plane.
- The unperturbed flow is in the negative x-direction.



$\phi(x, y, z; t)$ - **velocity potential at a point,** $u(x, y, t)$ **vertical displacement**- it can be large


Flutter Effect on an Airfoil

Goals and Challenges

- Eliminate the flutter, if possible.
- or control the flutter speed? Where? On the structure.
- post-flutter harvesting

Flutter - sustained oscillations. Can lead to a damage, breakage of the structure.

PDE Model. E.Dowell

Flow of gas in $D = R^3_+$

- $\rho_t + div(\rho v) = 0, in, D = R^3_+$ **Mass Eq.**
- $\rho[v_t + (v, \nabla)v] + \nabla p = 0 \; in \; D$ - **Momentum Eq**.

Structure on $\Omega \in R^2$

- $u_{tt} - \gamma \Delta u_{tt} + \Delta^2 u - [F(u), u] = p|_\Omega$ in Ω. **Large Oscillations.**

Here u represents the vertical displacement of the structure, $\gamma \geq 0$ represents possible rotational inertia , $F(u) = [\mathcal{F}(u) + F_0, u]$ where $\mathcal{F}(u)$ is Airy's stress function , $[u, w]$ v-Karman bracket. It models **large displacements** .

Linearizing the flow around **unstable** profile $[U, 0, 0]$, **un-viscous**, irrotational $[v = \nabla\phi]$, barotropic flow $[p = \alpha\rho]$
Flow of gas in $D = R^3_+$

- (1) $[\partial_t + U\partial_x]\rho + div(v) = 0,$
- (2) $(\partial_t + U\partial_x)[v_t + U\frac{\partial v}{\partial x} + \nabla p] = 0 \ in \ D$

irrotational $[v = \nabla\phi]$, barotropic flow $[p = \alpha\rho]$

$$(2) \to (3) \equiv (\partial_t + U\partial_x)[\phi_t + U\frac{\partial \phi}{\partial x} + \alpha\rho] = 0 \ in \ D$$

$$(1) + (3) = (\partial_t + U\partial_x)[\phi_t + U\frac{\partial \phi}{\partial x}] - \alpha\Delta\phi = 0 \ in \ D$$

Aeroelastic potential Ψ.

$$\Psi \equiv (\partial_t + U\partial_x)\phi, \quad v = \nabla\phi$$

Flow in $D \in R^3$ Hyperbolic system.

- $(\partial_t + U\partial_x)\phi = \Psi$
- $(\partial_t + U\partial_x)\Psi - \Delta\phi = 0$

taking $\alpha = 1$.

Structure on $\Omega \in R^2$ Hyperbolic like scalar eq.

- $u_{tt} - \gamma\Delta u_{tt} + \Delta^2 u - [F(u), u] = \Psi|_\Omega$ in Ω.

BOUNDARY CONDITIONS

For the Flow: R^2

- [Flow-tangency-Neumann:] $\frac{\partial \phi}{\partial \vec{n}} = \begin{pmatrix} (\partial_t + U\partial_x)u & \Omega \\ 0 & outside \end{pmatrix}$
- [Kutta Joukovsky:] $\frac{\partial \phi}{\partial \vec{n}} = \begin{pmatrix} (\partial_t + U\partial_x)u & \Omega \\ \Psi = 0 & outside \end{pmatrix}$

For the Structure: $\partial\Omega$

- [Clamped:] $u = \nabla u = 0, on\ \partial\Omega$
- [Free:] $D_n \Delta u + (1-\nu) D_\tau D_n D_\tau u = \Delta u + (1-\nu)[D_\tau^2 - div \vec{n} D_n]u = 0\ on\ \partial\Omega$
- [Hinged:] $u = \Delta u + k D_\tau^2 u = 0,\ on\ \partial\Omega.$

Bridge : Flow Neumann + Structure Free/hinged
Flag: Flow Neumann + Structure Free/Clamped
Oscillating Panel: Flow Neumann + Structure Clamped
Wing : Flow KJ + structure free/hinged

Question: formulate pde question pertinent to this flutter problem?

- Experiments [NASA, AFOSR Lab at UCLA. and Duke] show **"stabilizing "effect of the flow on the structure.**
- The model has no active dissipation.

Answer: Guided by the **experiment**: Study the following:

- **Generation** of a nonlinear evolution. Existence of dynamical system in the "finite energy space".
- Uniform **attraction** of structural evolution to a **finite dimensional set.**
- Strong **stability** to equilibria. $U < 1$
- **Control** of the resulting finite dimensional dynamical system.

Wellposedness

What has been known:

- **Global wellposedness of weak solutions** for $\gamma > 0$ only.

$$u_{tt} - \gamma \Delta u_{tt} + \Delta^2 u - [F(u), u] = \Psi|_{\Omega}, \; in \; \Omega$$

  Flow

$$(\partial_t + U\partial_x)\Psi - \Delta\phi = 0, in \; D$$

  with Neumann data for the flow and the structure clamped.
  Global unique weak solutions in

$$(\phi, \Psi, u, u_t) \in H^1(D) \times L_2(D) \times H_0^2(\Omega) \times H_0^1(\Omega)$$

  Boutet de Monvel, Chueshov [Doklady 1997, CRAS 1996, MMAS 1999] .I. Chueshov, A. Rezounenko (CRAS 1995, CMPDE 1997, CPAA 2015.) I. Ryzhykova (Comm. Math.Physics 2007, JMAA 2004)[**strongly damped** plates] .

- **No stabilizing effect shown by the flow. Why?**

. **Is this a wrong model?** A formal argument when $\gamma > 0$.

Rotational model: $\gamma > 0$ **Initial data for the flow are compactly supported.**

$u_{tt} - \gamma\Delta u_{tt} + \Delta^2 u - [F(u), u] = \boldsymbol{\Psi_U}$ **becomes after a long time**

$$u_{tt} - \gamma\Delta u_{tt} + \Delta^2 u - [F(u), u] = -(\partial_t + \mathbf{U}\partial_x)\mathbf{u} + \mathbf{q(u, t.x.y)}$$

$$q(u,t,x,y) \equiv \int_0^{t^*} ds \int_0^{2\pi} d\theta D^2(u(x-(U+sin\theta)s, y-scos\theta, t-s), DELAY$$

$$u_{tt} - \gamma\Delta u_{tt} + \underbrace{u_t}_{does\ \ not\ \ stabilize} + \Delta^2 u - [F(u), u] = \underbrace{-Uu_x + q(u,t,x,y)}_{destabilizes}$$

$$u_{tt} - \gamma\Delta u_{tt} + \underbrace{u_t - \gamma\Delta u_t}_{stabilizes} + \Delta^2 u - [F(u), u] = \underbrace{-Uu_x + q(u,t,x,y)}_{destabilizes}$$

Conclusion: To stabilize the structure : needs to add Δu_t **. Flow does not harvest such term. Flow does not stabilize the rotational model.**

Hidden stabilizing effect of the flow.

$$u_{tt} + \Delta^2 u - [F(u), u] = \mathbf{\Psi_U}$$

becomes after some time a nonlinear PDE with delay

$$u_{tt} + \Delta^2 u - [F(u), u] = -(\partial_{\mathbf{t}} + \mathbf{U}\partial_{\mathbf{x}})\mathbf{u} + \mathbf{q}(\mathbf{u}, \mathbf{t}, \mathbf{x}, \mathbf{y})$$

$$q(u,t,x,y) = \int_0^{t^*} ds \int_0^{2\pi} d\theta D^2(u(x-(U+sin\theta)s, y-scos\theta, t-s), DELAY$$

$$u_{tt} + \underbrace{u_t}_{stabilizes} + \Delta^2 u - [F(u), u] = -\underbrace{Uu_x + q(u,t,x,y)}_{destabilizes}$$

We will work with **irrotational** model

$$(\phi, \psi, u, u_t) \in H^1(D) \times L_2(D) \times H_0^2(\Omega) \times L_2(\Omega)$$

Loss of one derivative in structural velocity.

Structure-Final model

Structural equation-$u(t,x,y)$

$$u_{tt} + \Delta^2 u - [\mathbf{F}(\mathbf{u}), \mathbf{u}] = p_0 + (\partial_t + U\partial_x)\phi\big|_{\Omega} \ in \ \Omega$$
$$Boundary \ conditions : u = \nabla u = 0 \ on \ \partial\Omega$$

Initial conditions: $u(0) = u_0 \in H_0^2(\Omega), u_t(0) = u_1 \in L_2(\Omega)$

$p_0 \in L_2(\Omega)$ is the static aerodynamic pressure on the plate surface.
aeroelastic potential $= \Psi_U = (\partial_t + U\partial_x)\phi|_{\Omega}$,

Flow

Flow equation -$\phi(t, x, y, z)$

$$(\partial_t + U\partial_x)^2\phi = \Delta\phi$$

$$BC: \text{ on } \Omega: \ \partial_\nu \phi\big|_{z=0} = -(\partial_t + U\partial_x)u(x,y)) \text{ on } \Omega$$

$$\text{Outside } \Omega: \partial_\nu \phi\big|_{z=0} = 0$$

$$\text{Initial conditions}: \ \phi(t=0) = \phi_0, \ \phi_t(t=0) = \phi_1$$

- aeroelastic potential $= \Psi_U = (\partial_t + U\partial_x)\phi|_\Omega$,
- downwash $= (\partial_t + U\partial_x)u$

Von Karman -Airy Stress function nonlinearity

- $[F(u), u] = [u, \mathcal{F}(u) - F_0]$. F_0 is in-plane loading .
- $[g, h] = g_{xx}h_{yy} + g_{yy}h_{xx} - 2g_{xy}h_{xy}$ is the von Karman bracket.
- $\mathcal{F}(u)$ is the Airy Stress function, solves

$$\begin{cases} \Delta^2\mathcal{F}(u) = -[u,u] & \text{in } \Omega \\ \mathcal{F}(u) = \frac{\partial \mathcal{F}(u)}{\partial \nu} = 0 & \text{on } \Gamma \end{cases}$$

- $[F(u), u]$ cubic and *nonlocal* . $\mathcal{F}(u) : H^2(\Omega) \to H^{3-\epsilon}(\Omega)$ implies $[F(u), u] : H^2(\Omega) \to H^{-\epsilon_1}(\Omega)$, $\epsilon_1 > 0$, P.Ciarlet, J.L.Lions 2ϵ missing. **However**
- $\mathcal{F}(u) : H^2(\Omega) \to W^{2,\infty}(\Omega)$. $D^2\mathcal{F}(u)$ becomes a multiplier. IL, D. Tataru 1997 .

Goals:

Wellposedness of finite energy solutions.

Long time asymptotic behavior

Expectations: based on experimental studies:

- Elimination of the **flutter at the subsonic level** $U < 1$
- Asymptotic **reduction of structural dynamics to a finite dimensional attracting sets** (chaotic). **Dispersion provides a stabilizing effect**. Quantitize.

Mathematical challenges: Due to the loss of one derivative in u_t.

- lack of active dissipation on the flow and the structure;
- lack of compactness/regularity;
- potential degeneracy of the energy function.

Model: Energies

- **Plate** $\mathcal{E}_{pl}(t) = \frac{1}{2}(||u_t||^2_\Omega + ||\Delta u||^2_\Omega + 1/2||\Delta \mathcal{F}(u)||^2_\Omega - ([u, F_0], u)_\Omega)$

- **Flow**: $\mathcal{E}_{fl}(t) = \frac{1}{2}(||\phi_t||^2_D + ||\nabla\phi||^2_D - U^2||\partial_x \phi||^2_D)$

- **Interactive**: $\mathcal{E}_{int}(t) = 2U < u_x, \phi >_\Omega$

$$\mathcal{E}_{pl}(t) + \mathcal{E}_{fl}(t) + \mathcal{E}_{int}(t) = Constant$$

Total Energy -why it is interesting?

Balance of Energy: $\mathcal{E}(t) = \mathcal{E}(s)$. Two issues:
1. no dissipation, **2. loss of ellipticity** for $U \geq 1$.

Hidden dissipation - dispersive effects to account for.

- $U < 1$, $\mathcal{E}_{fl} = \frac{1}{2}\left(||\phi_t||^2 + ||\nabla\phi||^2 - U^2||\partial_x\phi||^2\right) \sim ||\phi_t||^2 + ||\nabla\phi||^2$
- $U = 1$, $\mathcal{E}_{fl} \sim ||\phi_t||^2 + ||\partial_z\phi||^2 + ||\partial_y\phi||^2 + 0 \cdot ||\partial_x\phi||^2$
- $U > 1$, $\mathcal{E}_{fl} \sim ||\phi_t||^2 + ||\partial_z\phi||^2 + ||\partial_y\phi||^2 - (U-1)||\partial_x\phi||^2$

Conclusions

1. flow does not stabilize the **rotational model** -confirming the experiment.
2. **Rotational model** yields smoother solutions by adding ONE space DERIVATIVE to u_t -making $f(u)$ compact on the energy space. Simplifies wellposedness analysis **however not physical**.
3. The original model brings aboard **new mathematics:**
   - $[\mathcal{F}(u), u]$ becomes critical
   - non dissipative, lack of gradient structure : Uu_x
   - PDE dynamics with the delay: $q(u)$ and its destabilizing effects.
   - Lack of compactness, regularity.

Gives rise to NEW Techniques

- PDE, harmonic and microlocal analysis,
- Dynamical systems theory for non-dissipative systems.

Main Results- Overview

- Existence and Hadamard wellposedness of **finite energy solutions** .
- Solutions are bounded for ALL TIMES. - CRITICAL role of **nonlinearity.** (**False for the linearization**)
- **Subsonic case:** with the **feedback control damping** implemented on the structure all solutions stabilize to the **stationary states.**

Conclusion: Flutter can be eliminated

- $U \neq 1$ **:** With the flow data compactly supported all weak solutions of the structure **without any dissipation** converge uniformly to a **finite dimensional and smooth set** .

Conclusion: Structural dynamics becomes finite dimensional asymptotically.

Nonlinear Semigroup -Generation

Theorem (Nonlinear semigroup)

- *Flow-structure interaction generates a continuous nonlinear semigroup*

$$S_t : H \to H = H_0^2(\Omega) \times L_2(\Omega) \times H^1(D) \times L_2(D)$$

- *Semigroup S_t is* **bounded** *for all $t > 0$ ($U < 1$).*
- *For compatible and suitably smooth initial data the corresponding solutions are* **smooth and global**.

Chueshov, I.L, Webster 2013 JDE

Remark: Solutions for **smooth** data with $U < 1$ in Boutet de Monvel, I.Chueshov, CRAS 1996, MMAS 1999.

Main Result - Subsonic and Supersonic

Theorem (Finite dimensional attracting set)

- *Let $U \neq 1$. Consider plate solutions in $H_{pl} = H_0^2(\Omega) \times L_2(\Omega)$. Then, there exists a compact set $\mathcal{A} \in H^3 \times H^2 \subset H_{pl}$ of finite fractal dimension such that*

$$lim_{t\to\infty} dist\{(u(t), u_t(t)), \mathcal{A}\} =$$

$$lim_{t\to\infty} inf_{u_0,u_1\in\mathcal{A}}[||u(t) - u_0||^2_{2,\Omega} + ||u_t(t) - u_1||^2_{0,\Omega}] = 0$$

for all **compactly supported initial conditions** *corresponding to the flow.*

Chueshov, I.L, J. Webster, CMPDE 2016, Oberwolfach Seminars 2018.

Remark $\mathcal{A}$ can be **chaotic** :limit cycles and periodic orbits. , E. Dowell [Duke] , J.Howell [CMU]

Strong Stability- Subsonic case

$\mathcal{N}$ denotes stationary solutions. Assume it is **finite.** (generically true. Can be eliminated :Haraux, Lojasiewicz lemma:)
Structural equation with a feedback control:
$u_{tt} + k(u_t) + \Delta^2 u - [F(u), u] = (\partial_t + U\partial_x)\phi|_\Omega$

Theorem (Stability, $U < 1$ and $k > k_0 > 0$, I.L, Webster, SIMA)

Let $U < 1$. Then any weak solution with **compactly supported flow initial data** *stabilizes to a stationary set.*
There exist $(u_0, u_1, \Phi_0, \Phi_1) \in \mathcal{N}$ such that for all $R > 0$.

$$lim_{t\to\infty}||u(t) - u_0||^2_{2,\Omega} + ||u_t(t) - u_1||^2_{0,\Omega} = 0$$

$$lim_{t\to\infty}||\Phi(t) - \Phi_0||^2_{1,B(R)} + ||\Phi_t(t) - \Phi_1||^2_{0,B(R)} = 0$$

where $B(R)$ denotes a ball of radius R.

CONSEQUENCE: **Flutter can be eliminated by applying damping to the structure only. Nonlinear effects are critical**

Buckling and Convergence to two different non-trivial steady states

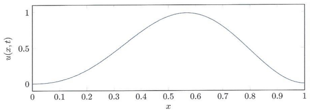


FIGURE 12. Plot of steady-state beam displacement for $U = 100$, $b = 50$, $b_0 = 1$, and varying k, nonlinear model. fig:f14 .

It is also possible to observe different choices of k to require convergence to different steady states. For $U = 100$, $b_0 = 1$, and $b = 100$, the choices $k = 1$ and $k = 2$ actually fig:f15 produce nontrivial steady states that are negatives of each other. In Figure 13 the midpoint fig:f16 displacement is plotted for these two cases, and the energies are give in Figure 14.

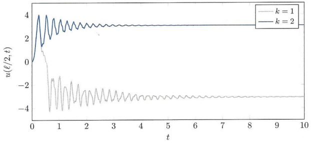


FIGURE 13. Plot of u at beam midpoint for $U = 100$, $b = 100$, $b_0 = 1$, and varying k, nonlinear model. fig:f15

Convergence to a limit cycle

0.6. **Convergence to a Limit Cycle.** In Figure 15, computed energies (log-scale) are plotted for a large flow velocity ($U = 5000$), significant in-axis compression parameter ($b = 50$), and $b_0 = 1$ for selected values of the damping parameter k. The sensitivity of the dynamics to the damping parameter can be seen by noting the relative rate of initial decay of energy as k increases. Plots of the beam midpoint displacement are given in Figure 16 - note the quick decay to the oscillatory limit cycle for larger values of k.

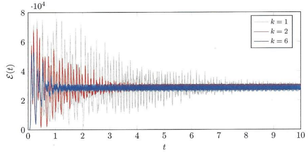


FIGURE 15. Plot of $\mathcal{E}(t)$ for for $U = 5000$, $b = 50$, $b_0 = 1$, nonlinear model. fig:f6

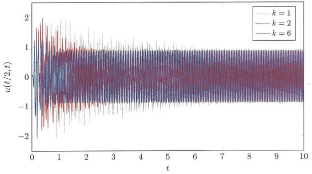


FIGURE 16. Plot of u at beam midpoint for $U = 5000$, $b = 20$, $b_0 = 1$, nonlinear model, varying k. fig:f7

It is possible to induce several different phenomena by manipulating parameter values. In Figure 17 the midpoint displacement is shown for $U = 5000$, $k = 100$,

Convergence to a limit cycle. Flutter

$b = 5000$, and $b_0 = 1000$. Note the initial transients are damped out quickly and the dynamics converges to a limit cycle.

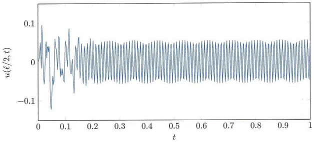


FIGURE 17. Plot of u at beam midpoint for $U = 5000$, $b = 5000$, $b_0 = 5000$, $k = 101$ nonlinear model.

fig:f8

Welllposedness (S_t, Y).

New "Supersonic" and K-J energy

$$E(t) = E_u(t) + E_\Phi(t)$$

$$E_u = 1/2 \int_\Omega (|u_t|^2 + |\Delta u|^2 + 1/2|\Delta \mathcal{F}(u)|^2 d\Omega - [u, F_0] u d\Omega$$

$$E_\Phi = 1/2 \int_D [|\Psi(t)|^2 + |\nabla \Phi(t)|^2] dD$$

Energy balance : $E(t) = E(s) - \mathbf{U} \int_s^t \int_\Omega \Psi|_\Omega u_x$

Control of **low** frequencies. For all $u \in H^2(\Omega) \cap H_0^1(\Omega)$ and all $\epsilon > 0$

- $||u||_{L_2(\Omega)} \leq \epsilon[||u||_{H^2(\Omega)} + ||\Delta \mathcal{F}(u)||_{L_2(\Omega)}] + C_{\epsilon,\Omega}$. This implies
- $||u||_{H^2}^2 \leq CE_u + M$. **Potential energy bounded from below.**

Good Energy, Bad Energy Balance . Microlocal Analysis arguments to handle bdry integral.

Long Time behavior

"Attractor" for the structure [plate] $U \neq 1$

Theorem (Existence of the structural attracting set.)

- *Let* $U \neq 1$. *Consider plate solutions in* $H_{pl} = H_0^2(\Omega) \times L_2(\Omega)$. *Then, there exists a compact set* $\mathcal{A} \in H_{pl}$ *of finite fractal dimension such that*

$$lim_{t\to\infty} dist\{(u(t), u_t(t)), U\} =$$

$$li_{t\to\infty} inf_{u_0,u_1\in\mathcal{A}}[||u(t) - u_0||^2_{2,\Omega} + ||u_t(t) - u_1||^2_{0,\Omega}] = 0$$

for all **compactly supported initial conditions** *corresponding to the flow.*

- *The said "attractor " is smooth* $\mathcal{A} \subset H^3(\Omega) \times H^2(\Omega)$.

Chueshov, I.L. Webster : CMPDE, Oberwolfach Seminars, 2018.

The analysis reduced to a finite dimensional invariant set
Determination of fluttter speed. Outline of the proof-below.

Hidden stabilizing effect of the flow

$$u_{tt} + \Delta^2 u = [F(u), u] + p(u,t,x,y)$$

$$p(u,t,x,y) \equiv -(\mathbf{u_t} + \mathbf{U u_x}) + q(u,t,x,y)$$

For $t >> T_0$,

$$u_{tt} + u_t + \Delta^2 u = [F(u), u] - U u_x + q(u,t,x,y)$$

$$q(u,t,x,y) = \int_0^{t^*} ds \int_0^{2\pi} D^2(u(x-(U+sin\theta)s, y - scos\theta, t-s)d\theta$$

$$D^1 = e^{-i\theta} \cdot \nabla^{\perp}_{x,y} = sin\theta \frac{\partial}{\partial x} + cos\theta \frac{\partial}{\partial y}$$

$$t^* = inf\{t, \vec{x}(U,\theta,s) \notin \Gamma, \vec{x} \in \Gamma\}, \vec{x} \equiv (x - s(U+sin\theta), y - scos\theta)$$

Difficulties:

1. **Nondissipative system,** Uu_x. **Low** frequencies.
2. **Lack of compactness** , $[F(u), u], q(u, t, x, y)$. **High** frequencies.

Strategy: I. Chueshov, I.L. Memoires AMS, 2008.

- **Absorbing** invariant set $B(R)$. Lyapunov function accounting for the delay term and good control of low frequencies. [maximum principle for nonlinear elliptic problems]

$$|u|_{H^s} \leq \frac{\epsilon}{U}[|u|_{H^2} + |\Delta\mathcal{F}(u)|_{L_2}] + C_\epsilon, \quad s < 2$$

  [**nonlinearity** cooperates [**talks to**] with the blow up-controlling low frequencies]

- **Sharp regularity** of Airy;s stress: [IL. D.Tataru, 1998]

$$||\mathcal{F}(u)||_{W^{2,\infty}} \leq C|u|^2_{H^2}$$

  Previous results [J.L.Lions,Dautray] not good

$$||\mathcal{F}(u)||_{H^{3-\infty}} \leq C|u|^2_{H^2}$$

  Note $H^{3-\epsilon}$ **does not embed** in $W^{2,\infty}$ which is a right space of multipliers for von Karman brackets.

- **Asymptotic smoothness -compensated compactness** criterion for a dynamical system (S_t, Y) . For all $\epsilon > 0$, for any bounded positively invariant B there exists $T(\epsilon, B)$

$$d_Y(S_T(y_1) - S_T(y_2)) \leq \epsilon + R_{\epsilon,B,T}(y_1, y_2), \;\; y_i \in B$$

where $R_{\epsilon,B,T}$ is a functional defined on $B \times B$:

$$\lim_m inf \lim_n inf \;\; R_{\epsilon,B,T}(y_n, y_m) = 0, \forall y_n \in B(R)$$

Then, (S_t, Y) is an **asymptotically smooth** dynamical system.

- With $z = u^1 - u^2$ with $y^i = (u^i, u^i_t, u^{t,i})$ and $y^i(t) \in B(R)$. For any $\epsilon > 0$ there exists $T = T_\epsilon(R)$ such that.

$$E_z(T) + \int_{T-t*}^{T} ||z(s)||^2_{H^2} ds \leq \epsilon + R_{\epsilon,T,R}(y^1, y^2)$$

$$\lim_m inf \lim_n inf \;\; R_{\epsilon,T,R}(y_n, y_m) = 0, \forall y_n \in B(R)$$

  Critical role of the structure [**symmetry**] of Karman bracket, **sharp regularity of Airy's** function and compensated compactness in **delay term**.

- **Existence of compact attractor** $\mathcal{A}$.
- **Quasistabilty estimate** obtained on the **attractor** $\mathcal{A}$ implies **finite dimensionality and smoothness**.
  I.L. Chueshov, V.K Evolutions,Springer, 2010.

$$||S_t(y_1) - S_t(y_2)||^2_Y \leq Ce^{-\omega t}||y_1 - y_2||^2_Y + LOT(S_t(y_1) - S_t(y_2))$$

  LOT lower order with at least **quadratic** order of homogenity.

Back to the equation: The key contribution of the nonlinear critical term. Exploiting the **symmetry and structure of vK bracket**

$$([\mathcal{F}(u^1), u^1] - [\mathcal{F}(u^2), u^2], z_t)_\Omega = \frac{d}{dt} Q(z) + P(z)$$

. where $Q(z)$ is of lower order [compact] so the term can be integrated in time. Instead, $P(z)$ remains critical but it has nice structure:

$$P(z) \le C_R |z|^2_{H^2} [|u^1_t|_{L_2} + |u^2_t|_{L_2}]$$

Relying on the obtained compactness of the attractor, sharp Airy's estimates one develops local estimates around points $e_j \in H^2(\Omega)$ close to $u^1_t, u^2_t \in L_2(\Omega)$.

- **Conclusion** taking the projection of the delayed dynamics on the plate dynamics gives the attracting set for the structure which is finite dimensional and smooth in $H^3(\Omega) \times H^1(\Omega)$.

Next: STRONG stability of the full system:flow and structure:
$U < 1$ **-subsonic**
Elimination of the flutter with a feedback control $k(u_t)$ on the structure.
STRATEGY .

1. **STEP 1:** Strong convergence to the set $\mathcal{N}$ of orbits driven by **smooth structural** data
   - Use **dispersion** estimates for the flow driven by the initial conditions
   - Analysis of the coupling via Neumann map: "tour de force " loosing derivatives . **Strong stability for smooth** initial data. $Y = (\Phi, \Phi_t, u, u_t)$.

$$S_t(Y_r) \to \mathcal{N}, Y_r \in D(A)$$

2. **STEP 2:** $k > 0$ Uniform Hadamard sensitivity uniformly in $t > 0$ when $||Y_r - Y|| \leq \epsilon$.

$$||S_t(Y_r) - S_t(Y)|| \leq \epsilon c(\int_0^t ||u_t||)$$

   Controlling the **rate of attraction** $||u_t|| \in L_1$**?.** We only know $||u_t|| \in L_2$.

$$|Y(t) - Y_m(t)^2|_Y \leq Ce^{|d(t)|_{L_1}}|Y(0) - Y_m(0)|_Y^2$$
$$d(t) = |u_t(t)|_{Y_{pl}} + |u_{m,t}(t)|_{Y_{pl}} \in L_2(0, \infty)$$

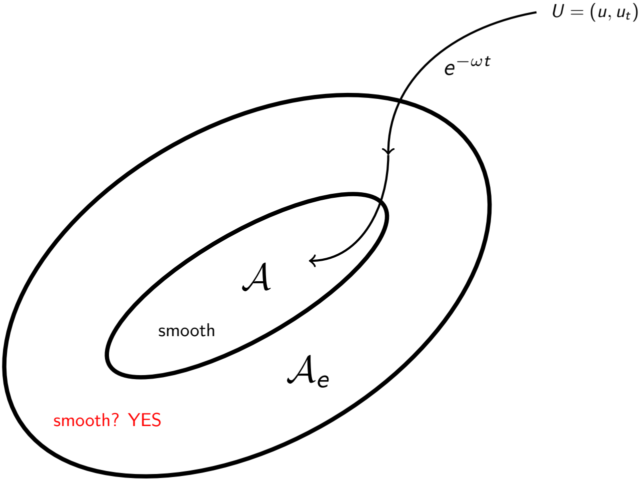
- **STEP 3 :** $k \geq 0$ Back to the structure. Construct **attractor** $\mathcal{A}$ for the structure only.
  Big Gun. **No dissipation, no compactness**. But "hidden" dissipation harvested from the flow and "hidden" smoothness. Smooth attracting set in $H^3 \times H^2$ was obtained in the previous theorem.
- **STEP 4:**
  - either $U(t)$ enters $\mathcal{A}$ -OK since smooth -go to Step 1.
  - or approaches $\mathcal{A}$. Question: **at which rate?**

- **STEP 5,** $k \geq 0$: Prove an existence of exponential attractor $\mathcal{A}_e \supset \mathcal{A}$.

$$dist(U(t)B, \mathcal{A}_e) \leq ce^{-\omega t}$$

  The rate is OK, but **smoothness???**. Difficulty: $\mathcal{A}_e$ is only positively invariant.

- **STEP 6,** $k \geq 0$; Prove smoothness of the exponential attractor $\mathcal{A}_e$. Using **quasi stability estimate** for a suitable decomposition of the flow which filters out initial data (Zelik, Vishik). **Attraction at the** L_1 **rate to a smooth set.** Go back to Step 1.



$$\text{dist}(U(t), \mathcal{A}_e) \leq Ce^{-\omega t}$$

Back to the flow

- Flow provides **"hidden" dissipation**
- Structure - "plate" provides **"hidden" asymptotic regularity on the attracting set**.
- **Propagating** these properties through the entire system -main challenge of the problem.

SUBSONIC CASE:

Flutter can be eliminated by applying damping to the structure.

SUPERSONIC CASE:

Flow **has stabilizing effect**.With **no damping** on the structure solutions are driven to a finite dimensional set. PDE dynamics reduced to ODE dynamics. Structure of the set : **chaotic, periodic orbits, limit cycles**

Conclusion

Stabilizing effect of the flow exhibited only for a correct model -unregularized and without rotational inertia .

Work in Progress and Open Problems

- **TRANSONIC** CASE. [$U = 1$]. Numerical evidence of shocks . Analysis **must account for nonlinearity of the flow**.
- **Kutta Jukovsky** boundary conditions.

$$\Psi = \phi_t + U\phi_x = 0 \quad off \quad the \quad wing.$$

  Mathematical interest: **"invertibility" of finite Hilbert transforms. L_p theory for $p \neq 2$.**

- **Free -clamped** boundary conditions on the plate. Tacoma bridge. Joint work with Filippo Gazzola and Denis Bonheur.
- Structure represented by **shell** model.
- **Nonlinear Flow equation**: NS or Euler

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THANK YOU