# A new criterion for bifurcation of homoclinic solutions for parameterized ordinary differential equations.

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In this talk, we will show how to study the following differential system:

$$\begin{cases} \dot{x}(t) = f(t, x(t), \lambda), \\ \lim_{t \to \pm\infty} x(t) = 0, \end{cases} \tag{1}$$

where $x \in W_0^{1,\infty}(\mathbb{R}, \mathbb{R}^d)$, $f : \mathbb{R} \times \mathbb{R}^d \times \Lambda \to \mathbb{R}^d$ is a Carathéodory function and $f(t, 0, \lambda) = 0$ for all $(t, \lambda) \in \mathbb{R} \times \Lambda$. Since the last assumption implies that the pair $(\lambda, 0)$ is a solution of (1), we are interested in nontrivial solutions, i.e., solutions $(\lambda, x)$ with $x \neq 0$.

In particular, we will present how to establish the existence of the so-called bifurcation points for (1). Recall that a point $\lambda^* \in \Lambda$ is a bifurcation point for homoclinic solutions of (1), if there is a sequence $(\lambda_n, x_n) \in \Lambda \times W^{1,\infty}(\mathbb{R}, \mathbb{R}^d)$ such that $x_n \neq 0$ is a solution of (1) with $\lambda_n \to \lambda^*$ and $x_n \to 0$ in $W^{1,\infty}(\mathbb{R}, \mathbb{R}^d)$.

Our approach uses the concept of parity (a crucial tool and topological invariant in the abstract bifurcation theory of nonlinear Fredholm operators, as developed by Fitzpatrick, Pejsachowicz and Rabier) and the Evans function $E(\lambda)$, which was originally used in the stability analysis of travelling waves in evolutionary PDEs.

The talk is based on my joint work with Christian Pötzsche.

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