Global positive solutions for equations with regularly varying differential operator and their asymptotic behaviour

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Second order nonlinear equations of the form

$$(a(t)\Phi(x'))' + b(t)F(x) = 0, \quad t \geq t_0 \tag{1}$$

are considered, where the functions a, b, F are continuous, with a, b positive, $uF(u) > 0$ for $u \neq 0$, and the operator Φ is an increasing odd homeomorphism

$$\Phi : (-\rho, \rho) \to (-\sigma, \sigma), \quad 0 < \rho, \sigma \leq \infty.$$

which is regularly varying in zero of index $\alpha > 0$, that is for any $\lambda > 0$

$$\lim_{u \to 0} \frac{\Phi(\lambda u)}{\Phi(u)} = \lambda^{\alpha}.$$

In this talk we analyze the problem of existence of globally positive solutions for (1) for a large variety of operators, for which the homogeneity property fails, extending some recent ones in [4, 5, 6]. The problem is related to the existence of radial solutions for PDEs with operators in divergence form. Boundary value problems associated to (1) or, more generally, to partial differential equations whose radial solutions satisfy (1), have been investigated by many authors, see for instance [1, 2, 3] and references therein.

In the second part of the talk, also the asymptotic behavior of such global solutions will be described, illustrating how a certain proximity between (1) and some auxiliary half-linear equations holds. Our method is based on an abstract fixed point result for invertible operators and on asymptotic properties of auxiliary half-linear differential equations. In particular, the concept of principal solutions and some comparison results play a fundamental role in finding good a-priori bounds for solutions.

References

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