Long-time asymptotics for nonlinear Schrödinger equations in one space dimension

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Many physical wave phenomena are described, to leading order, by linear dispersive equations. Examples arise in water waves, optics, plasma physics, and elasticity. In a dispersive medium, waves of different wavelengths travel at different speeds, so a localized disturbance typically spreads out and decreases in amplitude over time. A natural question is whether this description remains accurate over long times when nonlinear effects are taken into account.

We study this question for nonlinear Schrödinger equations on the real line with polynomial nonlinearities. For small, spatially localized initial data, solutions exist for all positive time and decay at the same rate as solutions of the linear equation. Their asymptotic behavior, however, differs from the linear one already at leading order, due to resonant self-interaction: a phenomenon known as modified scattering.

I will explain this mechanism and then, if time allows, discuss how to go beyond the leading-order approximation. The proof builds on a method introduced by Kato and Pusateri in a related setting. This is joint work with Tony Salvi.

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