Institute of Mathematics, Lodz University of Technology

International Meetings on Differential Equations and Their Applications

March 3 – June 23, 2021, Łódź, Poland

We are pleased to invite you to participate at the International Meetings on Differential Equations and Their Applications, to be held on Wednesdays at 10:15 (Warsaw time) from March 3 to June 23, 2021.

The meetings are organized as an online event via Zoom.

Organizers:

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Schedule of meetings

March 3, 2021	10:15 AM	**Dominika Pilarczyk** Wroclaw University of Science and Technology, Poland Stability of singular solutions to the Navier–Stokes system
March 10, 2021	10:15 AM	**Urszula Foryś** University of Warsaw, Poland Mathematical modeling of prostate cancer (PC) treatment
March 17, 2021	10:15 AM	**Jean Mawhin** Université Catholique de Louvain, Belgium Gauss, Bolzano and Cauchy meet in the complex field
March 24, 2021	10:15 AM	**Torsten Lindström** Linnaeus University, Sweden Destabilization, stabilization and multiple attractors in saturated mixotrophic environments
March 31, 2021	10:15 AM	**Meirong Zhang** Tsinghua University, China Stability for Bifurcated, Nonconstant, Symmetric Periodic Orbits of the Elliptic Sitnikov Problem
April 7, 2021	10:15 AM	**Robert Stańczy** University of Wroclaw, Poland Existence and Multiplicity Results for Dirichlet Problem with Fractional Laplacian and Nonlinearity
April 14, 2021	10:15 AM	**Gennaro Infante** Università della Calabria, Italy Nontrivial solutions of systems of perturbed Hammerstein integral equations with functional terms
April 21, 2021	**6:00 PM**	**Grzegorz Rempala** Ohio State University, USA Mathematical Models of Epidemics: Aggregating Stochastic Dynamics on Random Graphs
April 28, 2021	10:15 AM	**Christian Pötzsche** Universität Klagenfurt, Austria Numerical Dynamics of Integrodifference Equations: Local behavior

May 12, 2021 10:15 AM **Joanna Janczewska**

Gdansk University of Technology, Poland

Homoclinics for singular strong force Lagrangian systems

May 19, 2021 10:15 AM **Mirosław Lachowicz**

University of Warsaw, Poland

Integro–differential equations and their applications in 4 fields and 5 scientific disciplines

May 26, 2021 10:15 AM **José Ángel Cid**

University of Vigo, Spain

The averaging method for semilinear equations and some applications

June 2, 2021 10:15 AM **Aleksandra Orpel**

University of Lodz, Poland

Sub–supersolution method for singular elliptic problems

June 9, 2021 10:15 AM **Tomasz Cieślak**

Polish Academy of Sciences, Poland

Time–optimal control of a two–peakon collision

June 16, 2021 10:15 AM **Pavel Drábek**

University of West Bohemia, Czech Republic

Remarks on Nonlinear Eigenvalue Problems

June 23, 2021 10:15 AM **George L. Karakostas**

University of Ioannina, Greece

Hyperexponential asymptotics

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Abstracts

The averaging method for semilinear equations and some applications

José Ángel Cid

We present an averaging method in the abstract setting of semilinear equations of the form

$$Lx = \varepsilon N(n, \varepsilon),$$

where L is a linear Fredholm operator of index zero.

Several applications to the existence of periodic solutions for both ordinary and functional differential equations and to the existence of solutions for some non–local boundary value problems are given.

The talk is based on the paper:
Cid, J. Á., Mawhin, J. and Zima, M., An abstract averaging method with applications to differential equations, J. Differential Equations 274 (2021), 231–250.

José Ángel Cid, University of Vigo, Spain
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Time–optimal control of a two–peakon collision

Tomasz Cieślak

I'm going to talk about a common paper with my PhD student, Bidesh Das. It concerns the time–optimal control of a collsion of two–peakon. Two–peakons are particular, important solutions of the Camassa–Holm equation. We shall obtain the time–optimal control with the use of Bellman's dynamic programming method.

Tomasz Cieślak, Polish Academy of Sciences, Poland
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Remarks on Nonlinear Eigenvalue Problems

Pavel Drábek

The eigenvalue problem for the p–Laplacian is a natural extension of the linear eigenvalue problem for the Laplace operator ($p = 2$). Courant–Weinstein variational principle provides the tool in order to identify complete set of eigenvalues and eigenfunctions in the linear case. In the nonlinear case ($p \neq 2$) there are also several variational methods which lead to the infinite sequence of eigenvalues (let us mention Ljusternik–Schniremann method among others) and allow for the study of the nodal structure of eigenfunctions analogous, e.g., to the Courant nodal domain theorem. However, to find the method which would lead to the complete set of all eigenvalues is a long standing open problem. The purpose of this talk is to show that even in the very special case of the eigenvalue problem for the p–Laplacian on a disk (in the plane) some unexpected properties of eigenfunctions and eigenvalues can occur with varying $p \in (1, \infty)$. In particular, we want to point out that the set of all eigenvalues and eigenfunctions for $p \neq 2$ is not merely a "continuous deformation" of the linear case $p = 2$ and that the properties of the nonlinear eigenvalue problem are much more involved and complicated.

Pavel Drábek, University of West Bohemia, Czech Republic
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Mathematical modeling of prostate cancer (PC) treatment

Urszula Foryś

I would like to present two models describing two types of PC treatment. Both models were proposed in cooperation with the team of prof. Zvia Agur from the Institute for Medical Biomathematics in Israel. First model described immunotherapy, which is a non–standard type of treatment, while second one is related to completely standard therapy for hormone sensitive PC. It occurred that in both cases asymptotic behavior could be approximated by one–dimensional dynamics.

First part of my presentation will be based on:
U. Foryś, M. Bodnar, Y. Kogan, Asymptotic dynamics of some t–periodic one–dimensional model with application to prostate cancer immunotherapy, Journal of Mathematical Biology 73 (4) 2016, pp. 867–883.
Second part is related to the results obtained within NAWA project (manuscript under review).

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Nontrivial solutions of systems of perturbed Hammerstein integral equations with functional terms

Gennaro Infante

We discuss the solvability of a fairly general class of systems of perturbed Hammerstein integral equations with functional terms that depend on several parameters. The nonlinearities and the functionals are allowed to depend on the components of the system and their derivatives. The results are applicable to systems of nonlocal second order ordinary differential equations subject to functional boundary conditions, this is illustrated in an example. Our approach is based on the classical fixed point index.

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Homoclinics for singular strong force Lagrangian systems

Joanna Janczewska

We study the existence of homoclinic solutions for a class of Lagrangian systems $\frac{d}{dt}\left(\nabla\Phi(\dot{u}(t))\right) + \nabla_u V(t, u(t)) = 0$, where $t \in \mathbb{R}$, $\Phi\colon \mathbb{R}^2 \to [0, \infty)$ is a G–function in the sense of Trudinger, $V\colon \mathbb{R} \times (\mathbb{R}^2 \setminus \{\xi\}) \to \mathbb{R}$ is a C^1–smooth potential with a single well of infinite depth at a point $\xi \in \mathbb{R}^2 \setminus \{0\}$ and a unique strict global maximum 0 at the origin. Under a strong force condition around the singular point ξ, via minimization of an action integral, we will prove the existence of at least two geometrically distinct homoclinic solutions $u^{\pm}\colon \mathbb{R} \to \mathbb{R}^2 \setminus \{\xi\}$.

Joanna Janczewska, Gdansk University of Technology, Poland
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Hyperexponential asymptotics

George L. Karakostas

The Hardy's log–exp composition scale is extended to hyperexponentials and by their use a growth index on a class of functions between normed spaces is defined. The growth index establishes a classification of these functions in levels fitted in the scale and some essential properties of it are presented. As applications, it is shown that, in the case of levels greater than 1, the growth index of the solutions of some Volterra integral equations cannot exceed the growth index of the perturbations, a fact which help to get some exponential stability results for ODEs. Finally, a BVP defined by a second order one–large–parameter ODE is considered and the growth index (with respect to the level 1) of the error between any solution and a C^1–approximate solution is estimated.

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Integro–differential equations and their applications in 4 fields and 5 scientific disciplines

Mirosław Lachowicz

I am going to show that blow–ups of solutions, that usually are treated as something bad, can in fact describe some self–organization phenomena, 'positive' (like healing) or 'negative' (like society polarization). Mathematically it is going to be the theory of integro–differential equations that is applied to processes in Social Sciences (opinion formation), Economics ('lemons and cherries' theory), Biology (DNA denaturation), Medicine (tendon healing process – collagen remodeling) and the redistribution in a lift.

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Destabilization, stabilization and multiple attractors in saturated mixotrophic environments

Torsten Lindström

The ability of mixotrophs to combine phototrophy and phagotrophy is now well recognized and found to have important implications for ecosystem dynamics. In this paper, we examine the dynamical consequences of the invasion of mixotrophs in a system that is a limiting case of the chemostat. The model is a hybrid of a competition model describing the competition between autotroph and mixotroph populations for a limiting resource, and a predator–prey-type model describing the interaction between autotroph and herbivore populations. Our results show that mixotrophs are able to invade in both autotrophic environments and environments described by interactions between autotrophs and herbivores. The interaction between autotrophs and herbivores might be in equilibrium or cycle. We find that invading mixotrophs have the ability to both stabilize and destabilize autotroph-herbivore dynamics depending on the competitive ability of mixotrophs. The invasion of mixotrophs can also result in multiple attractors.

Joint work with: Yuanji Cheng and Subhendu Chakraborty.

Reference:

T. Lindström, Y. Cheng, and S. Chakraborty, SIAM J. APPL. MATH. 80(6), 2338–2364, 2020.

Torsten Lindström, Linnaeus University, Sweden
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Sub-supersolution method for singular elliptic problems

Aleksandra Orpel

We describe the sub–supersolution approach for the following elliptic problem

$$\begin{cases} \triangle u(x) + f(x, u(x)) - b(x)u^{-\alpha}(x)||\nabla u(x)||^{\beta} + g(x)x \cdot \nabla u(x) = 0, \\ \lim\limits_{||x||\to\infty} u(x) = 0, \end{cases}$$

discussed in the exterior domain $\Omega_R = \{x \in \mathbb{R}^n, ||x|| > R\}$, where $n > 2$ and $0 < 2\alpha \leqslant \beta \leqslant 2$. Our main goal is to find conditions guaranteeing the existence of positive solutions u of our equation with the asymptotic decay $u(x) = O(||x||^{2-n})$ as $||x|| \to \infty$ and show that these solutions have finite energy in a neighborhood of infinity. Our approach allows us to investigate both sublinear and superlinear cases of f and neglect the radial symmetry of b and g.

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Gauss, Bolzano and Cauchy meet in the complex field

Jean Mawhin

The seminar is centered on the fundamental theorem of algebra (FTA) : *any nonconstant polynomial with complex coefficients has a complex root.*

After an introduction to the history of the theorem and to its proofs (among which the four ones of Gauss), we describe an existence theorem having as immediate consequences both the FTA and an extension of the Bolzano theorem to holomorphic functions. Its very simple proof is based upon simplest version of the Cauchy integral theorem.

This is how Gauss, Bolzano and Cauchy meet in the complex field.

Jean Mawhin, Université Catholique de Louvain, Belgium
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Stability of singular solutions to the Navier–Stokes system

Dominika Pilarczyk

This is a joint work with Marco Cannone, Grzegorz Karch and Gang Wu. We study asymptotic properties of solutions to the three dimensional Navier-Stokes system for incompressible fluid in the whole three dimensional space. We deal either with the Cauchy problem or with the stationary problem where solutions may be singular due to singular external forces which are either singular finite measures or more general tempered distributions with bounded Fourier transforms. We present results on asymptotic properties of such solutions either for large values of the space variables (so called the far-field asymptotics) or for large values of time. We show that every solution from $C_w\Big((0,\infty);\mathcal{PM}^2\Big)$ behaves, for large values of $|x|$ (which is measured by the Lebesgue norm $L^q(\mathbb{R}^3)$ with $q \in [2,3)$), as a solution to the heat equation. Moreover, we consider stationary solutions and we describe their stability in the norms of $L^q(\mathbb{R}^3)$ with some $q < 3$.

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Numerical Dynamics of Integrodifference Equations: Local behavior

Christian Pötzsche

Integrodifference equations are popular models in theoretical ecology to describe the temporal evolution and spatial dispersal of populations having nonoverlapping generations. This talk addresses the question which properties of such infinite–dimensional dynamical systems persist under numerical discretization i.e. in actual computational simulations. Here we focus on local properties such as the saddle point structure consisting of stable and unstable manifolds near hyperbolic solutions in an increasingly more general setting. Besides persistence we will also establish convergence preserving the order of the numerical method.

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Mathematical Models of Epidemics: Aggregating Stochastic Dynamics on Random Graphs

Grzegorz Rempala

Motivated by the classical Susceptible–Infected–Recovered (SIR) epidemic models and the data from current COVID–19 pandemic, we consider a class of stochastic dynamical systems (SDSs) evolving on random graphs. We show that the dynamics of such SDSs may be approximately described in terms of implicit survival functions and certain random measures. This survival interpretation allows us to employ tools from statistical theory of survival analysis to address various issues with data collection and statistical inference in classical SIR models. It also offers an alternative to more standard statistical methods based on the theory of hidden Markov processes. In particular, we propose and numerically validate a statistical inference procedure for SDS–likelihoods that is relying on observed marginal likelihoods generated by typically epidemic curves. Only a slightly more complected SDS model was successfully used by the state of Ohio to predict the amount of state COVID–19 burden in the early months of the 2020 pandemic. If time permits, I will also briefly outline the main ideas behind that specific model.

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Existence and Multiplicity Results for Dirichlet Problem with Fractional Laplacian and Nonlinearity

Robert Stańczy

The existence and multiplicity results for Dirichlet BVPs with the fractional Laplacian are established depending on the range of parameter and behavior of the nonlinearity at zero and at infinity.

References:
D. Bors, R. Stańczy, Existence and Multiplicity Results for Dirichlet Problem with Fractional Laplacian and Nonlinearity, submitted.
T. Kulczycki, R. Stańczy, Multiple solutions for Dirichlet nonlinear BVPs involving fractional laplacian, Discrete Continuous Dynamical Systems 19 (2014), 2581–2591.

Robert Stańczy, University of Wroclaw, Poland
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Stability for Bifurcated, Nonconstant, Symmetric Periodic Orbits of the Elliptic Sitnikov Problem

Meirong Zhang

The (circular and elliptic) Sitnikov problems (S_0) and (S_e) are the simplest restricted 3–body problem. A lot of nonconstant periodic solutions have been constructed in literature using different approaches.

In this talk, we are concerned with the families $\phi_m(t,e)$ and $\varphi_m(t,e)$ of symmetric (either odd or even in time), $2m\pi$–periodic solutions of (S_e), emanated from those of (S_0). Based on the theory for Hill's equations, we will develop some stability criteria for the eccentricity e small. Then we can give an analytical proof to the stability/instability for several families of symmetric periodic orbits for $0 < e \ll 1$. For example, all families of odd $4n\pi$–periodic orbits are unstable, while the families of even $(8n-4)\pi$–periodic orbits are stable and the families of even $8n\pi$–periodic orbits still unstable. Here n is an arbitrary integer.

The talk is based on some joint works with X. Cen, C. Liu, et al.

References:

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J. Galán-Vioque, D. Nuñez, A. Rivera, and C. Riccio, Stability and bifurcations of even periodic orbits in the Sitnikov problem, *Celestial Mech. Dynam. Astronom.*, 130:82 (2018), 20 pp.

J. Llibre and R. Ortega, On the families of periodic orbits of the Sitnikov problem, *SIAM J. Appl. Dyn. Syst.*, 7 (2008), 561–576.

W. Magnus and S. Winkler, *Hill's Equations*, John Wiley, New York, 1966.

R. Ortega, Symmetric periodic solutions in the Sitnikov problem, *Arch. Math. (Basel)*, 107 (2016), 405–412.

M. Zhang, X. Cen, and X. Cheng, Linearized stability and instability of nonconstant periodic solutions of Lagrangian equations, *Math. Methods Appl. Sci.*, 41 (2018), 4853–4866.

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