Positive periodic solutions to the system of nonlinear delay differential equations

Robert Hakl

Considering the system of nonlinear delay differential equations

$$u_i'(t) = -d_i(t)u_i(t) - H_i(t)u_i(t-\sigma_i(t)) + \sum_{j=1}^{n} a_{ij}(t)u_j(t-\nu_{ij}(t)) + \lambda_i \sum_{k=1}^{N} P_{ik}(t)u_i(t-\tau_{ik}(t))f_{ik}(u_i(t-\mu_{ik}(t))) \quad \text{for a. e. } t \in \mathbb{R} \quad (i=1,\dots,n) \tag{1}$$

with ω-periodic coefficients and parameters $\lambda_i \geqslant 0$ $(i=1,\dots,n)$, it is shown that (under suitable conditions) there exists a continuous $(n-1)$-surface $S \subset \mathbb{R}^n$ that splits the non-negative orthant of parameters $(\lambda_i)_{i=1}^n$ into two parts: the first part Λ_1 is bounded, contains the origin, and the only ω-periodic solution to the system (1) is a zero solution provided $(\lambda_i)_{i=1}^n \in \Lambda_1$; the second part Λ_2 is unbounded and it consists of parameters for which the system (1) has a positive ω-periodic solution.

Robert Hakl, Institute of Mathematics, Czech Academy of Sciences
e-mail : hakl@ipm.cz