On a range of exponents for absence of Lavrentiev phenomenon for double phase functionals

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joint work with Miroslav Buliček, Jakub Skrzeczkowski

For a class of functionals having the (p, q)–growth, namely:

$$\int_\Omega a(x)|\nabla_x u(x)|^q + |\nabla_x u(x)|^p \, dx$$

(for $\Omega \subset \mathbb{R}^d$ and $p < q$), we establish an improved range of exponents p, q for which the Lavrentiev phenomenon does not occur.

By Lavrentiev phenomenon we meen situation that minimum of the functional on the set of smooth functions is strictly biger to the minimum on some Sobolev space.

The proof is based on a standard mollification argument and Young convolution inequality. Our contribution is two–fold. First, we observe that it is sufficient to regularise only bounded functions. Second, we exploit the L^∞ bound on the function rather than the L^p estimate on the gradient.

Our proof does not rely on the properties of minimizers to variational problems but it is rather a consequence of the underlying Musielak–Orlicz function spaces. Moreover, our method works for unbounded boundary data, the variable exponent functionals and vectorial problems. In addition, the result seems to be optimal for $p \leqslant d$.

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