Optimal existence results for the Keller–Segel chemotaxis system

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Abstract
The minimal chemotaxis system consists of two parabolic equations

$$u_t - \Delta u + \nabla \cdot (u \nabla \varphi) = 0, \qquad \tau \varphi_t = \Delta \varphi + u,$$

and describes the evolution of the density population u under diffusion and attraction directed by gradient $\nabla \varphi$ of the chemoattractant density in the whole space $\mathbb{R}^d$. The question: What is the minimal regularity and what is the maximal size of initial data leading to global-in-time solutions? is addressed in the higher dimensional case $d \geqslant 3$. At the same time conditions for the finite time blowup of solutions are found. The answers depend in a sensitive way on the size of the coefficient τ varying from 0 (the parabolic-elliptic case) to ∞ in the fully parabolic case.

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