The method of solution-regions applied to existence and multiplicity problems for systems of first order differential equations with nonlinear boundary conditions

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Abstract: We present existence and multiplicity results for systems of first order differential equations of the form:

$$\begin{aligned} &u'(t) = f(t, u(t)) \quad \text{for a.e. } t \in [0, T], \\ &u \in \mathcal{B}; \end{aligned} \tag{1}$$

where $f : [0, T] \times \mathbb{R}^N \to \mathbb{R}^N$ is a Carathéodory function and $\mathcal{B}$ denotes a boundary value condition. No growth conditions will be imposed on f. Even though this problem was widely treated, few multiplicity results can be found in the literature.

First, we will present the method of solution-regions to establish existence and multiplicity results for the system (1) in the case where $\mathcal{B}$ denotes an initial value condition or a periodic boundary condition. A solution-region will be a suitable set R in $[0, T] \times \mathbb{R}^N$ for which we will deduce that it contains the graph of viable solutions. We will show that this method generalizes the methods of upper and lower solutions and of solution-tubes. We will introduce also the notion of strict solution-regions and we will give conditions insuring the existence of at least three viable solutions of (1). Many non trivial examples will be presented throughout this presentation to show that the method of solution-regions is a powerful tool to establish the existence of solutions of systems of differential equations.

Then, we will show how the method of solution-regions can be extended to systems of differential equations with nonlinear boundary conditions.

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