Numerical methods for nonlocal and nonlinear parabolic equations with applications in hydrology and climatology

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Many natural and industrial phenomena exhibit nonlocal behaviour in temporal or spatial dimension. The former is responsible for processes for which its whole history influences the present state. The latter, on the other hand, indicates that faraway regions of the domain may have some impact on local points. This is useful in describing media of high heterogeneity.

Partial differential equations that are nonlocal involve one or several integral operators that encode this behaviour. For example, Riemann-Liouville or Caputo derivatives are used in temporal direction, while fractional Laplacian or its relatives describe spatial nonlocality. When it comes to numerical methods the discretization of these requires more care than their classical versions. Moreover, it is usually much more expensive, both on CPU and the memory, to conduct simulations involving nonlocal equations.

In this talk we will present several approaches to discretize nonlocal and nonlinear parabolic equations. These include: transformation into a pure integral equation for the time-fractional porous medium equation and Galerkin spectral methods for a general parabolic equation with temporal nonlocality. We will prove stability and convergence of these methods illustrating all the theoretical results with numerical simulations implemented in Julia programming language with parallelization. The talk is based on [2, 3, 1, 4].

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Literatura

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