Fractional harmonic maps in homotopy classes

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We extend the seminal work of Sacks and Uhlenbeck into the fractional framework. We look for maps between manifolds $\mathcal{M}$ and $\mathcal{N}$ that minimize the critical Sobolev $W^{s,n/s}$ energy, where n is the dimension of the manifold $\mathcal{M}$ in the domain. We prove that in the case when the target manifold $\mathcal{N}$ has trivial n-th homotopy group then in every homotopy class of maps between an n-manifold $\mathcal{M}$ to $\mathcal{N}$ there exists a minimizing fractional harmonic map. In case when the n-th homotopy group of the target manifold is not trivial we prove that there exists a generating set for the n-th homotopy group of the target manifold $\mathcal{N}$ consisting of minimizing fractional maps.

We develop new tools which are interesting on their own, such as a removability result for point-singularities and a balanced energy estimate for non-scaling invariant energies.

Joint work with Armin Schikorra.

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