Quasilinear parabolic systems describing taxis and chemical signaling in predator-prey models.

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The following quasilinear parabolic system of equations is an extension of the classical o.d.e. Bazykin's model. It belongs to the class of pursuit-evasion models describing predator-prey interactions combining the prey-taxis (pursuit) as a foraging strategy of predators with repulsive chemotaxis as the defense strategy of prey understood as a directed movement of prey in the opposite direction to the density gradient of a chemical released by predator (evasion). Denoting the densities of the predator, prey and the chemical by $P, N, W : \Omega \times (0\,,\infty) \mapsto R$, respectively, the model reads

$$\begin{cases} P_t = d_P \Delta P - \nabla \cdot \left(\dfrac{\xi P \nabla N}{1+\sigma P} \right) + bF(N)P - \delta P - \delta_1 P^2\,, \\ N_t = d_N \Delta N + \nabla \cdot \chi N \nabla W - F(N)P + rN - r_1 N^2\,, \\ W_t = d_W \Delta W + \gamma P - \mu W\,, \end{cases}$$

defined in a bounded domain $\Omega \subset R^n$ with smooth boundary, supplemented with initial and homogeneous Neumann boundary conditions. The function F describes the consumption rate, r and δ are death rate coefficients, r_1 and δ_1 are intraspecific competition coefficients and b reflects the conversion of food into offspring in predators. The W-equation describes the spread of the signaling chemical which is released by the predators. The presence of cross-diffusion terms in the system, typical for pursuit-evasion models, leads to challenging analytical problems appealing a remarkable interest. The parameter $\sigma \geqslant 0$ is used to describe suppression of predators velocity due to increase of predators density. The existence of global-in-time classical solutions depends on parameter values and space dimension n. We prove that there is a critical value $\sigma = \sigma_c$, depending on values of other parameters, above which there exist a global-in-time solution provided $n \leqslant 3$. Numerical simulations indicate blow-up of solutions in finite time for $n = 2$ if $\sigma \in [0\,,\sigma_c)$ and initial data are suitably chosen, other simulations exhibit occurrence of pattern formation for some range of parameter values.

References:

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