Existence and Non-Existence of solutions of third order equations coupled to three-point boundary conditions.

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This is a joint work with Prof. Nikolay D. Dimitrov (University of Ruse, Bulgaria).

In this talk, we present some results, obtained in [1], of existence and non-existence of solutions of the third order nonlinear one-parameter differential equation

$$u'''(t) = -\lambda\, p(t)\, f(u(t)),\ \text{a.e. } t \in [0,1], \tag{1}$$

coupled to the three-point boundary value conditions

$$u(0) = 0,\ u''(\eta) = \alpha\, u'(1),\ u'(1) = \beta\, u(1), \tag{2}$$

with $0 \leqslant \alpha \leqslant 1, 0 \leqslant \beta < \frac{2}{2-\alpha}$ and $0 \leqslant \eta \leqslant \frac{1}{3}$.

We construct its related Green's function and prove that it changes sign on its square of definition, but it has (different) constant sign whenever $0 \leqslant s < \eta$ or $\eta < s \leqslant 1$.

By assuming suitable sign conditions on the nonlinear part of the equation and under additional conditions on the asymptotic behavior of function f, by defining suitable cones on $C^1([0,1])$, we deduce, for a particular set of values of the parameter λ, the existence of positive and increasing solutions on the whole interval of definition which are convex on $[0,\eta]$. The results hold by means of degree theory.

[1] A. Cabada, N. D. Dimitrov, *Third-order differential equations with three-point boundary conditions.* Open Math. **19** (2021), 1, 11–31.

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