Invariance and strict invariance for nonlinear evolution problems with applications

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This talk presents sufficient conditions for the invariance and strict invariance of closed sets with respect to evolution problems of the form

$$\dot{u} \in -Au + f(u), \qquad u(0) = x_0,$$

where A is a quasi m-accretive (possibly nonlinear) operator in a Banach space X and f is a continuous perturbation. The constraint set is given as a sublevel set $K = \{x : V(x) \leqslant 0\}$ of a locally Lipschitz functional V. The key idea is to use the Dini A-directional derivative $D_A V(x; f(x))$ and to require an estimate

$$D_A V(x; f(x)) \leqslant \omega(V(x)) \quad \text{for } x \in (U \setminus K) \cap \overline{D(A)},$$

where ω is a uniqueness function. Several variants of this criterion are given, including versions that do not require reflexivity of X and are well suited for parabolic PDEs and age-structured population models. In particular, we discuss applications to the nonlinear double obstacle problem with the p-Laplacian, to reaction-diffusion equations, and to the McKendrick model. The results extend earlier work of Cannarsa, Da Prato and Frankowska to nonlinear operators and to a non-reflexive setting, and they also provide new strict invariance criteria that guarantee that solutions immediately enter the interior of the constraint set.

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