A geometric method for infinite-dimensional chaos: symbolic dynamics for the Kuramoto-Sivashinsky PDE on the line

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We discuss a method for rigorous study of dynamics of dissipative PDEs on the torus (i.e. assuming periodic boundary conditions).

As an example we consider the Kuramoto-Sivashinsky PDE on the line

$$u_t = -\nu u_{xxxx} - u_{xx} + (u^2)_x, \qquad \nu > 0, \tag{1}$$

where $x \in \mathbb{R}$, $u(t,x) \in \mathbb{R}$ and we impose odd and periodic boundary conditions

$$u(t,x) = -u(t,-x), \qquad u(t,x) = u(t,x+2\pi). \tag{2}$$

and parameter $\nu = 0.1212$.

We give a computer-assisted proof of the existence of symbolic dynamics, countable infinity of periodic orbits with arbitrary large periods and homo- and heteroclinic connections between them. The proof combines geometric tools from dynamics with rigorous numerics for dissipative PDEs.

This is a joint work with Daniel Wilczak.

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