Conditions for uniform h–dichotomy in terms of uniform non criticality, expansiveness and via generalized Floquet theory

Gonzalo Robledo Veloso

Given a linear system of ordinary differential equations:

$$x' = A(t)x \quad \text{for any } t \in J := (a_0, +\infty),$$

it is said that the system has an h–**dichotomy** if any non–trivial solution $t \mapsto x(t)$ admits a unique decomposition $x(t) := x_+(t) + x_-(t)$ such that there exists $K \geqslant 1$ and $\alpha > 0$ and the following estimations are fulfilled for any $t \geqslant t0 > a_0$:

$$|x_+(t)| \leqslant K \left(\frac{h(t)}{h(t_0)}\right)^{-\alpha} \quad \text{and} \quad K^{-1} \left(\frac{h(t)}{h(t_0)}\right)^{\alpha} \leqslant |x_-(t)|,$$

where $h\colon J \to (0, +\infty)$ is an increasing homemorphism. In consequence, an h–dichotomy means that any non trivial solution can be decomposed in a contractive part $x_+(\cdot)$ and an expansive one $x_-(\cdot)$, which are dominated by the *growth rate* h.

In this talk, we carry out an study of the equivalences between the properties of h–dichotomy, h–noncriticality and h–expansiveness of a linear nonautonomous ODE system which had been initiated in a previous work. This work generalizes a result obtained by K.J. Palmer in 2006 for the exponential dichotomy, which corresponds to the case $h(t) = e^t$.

Moreover, we extend a result of the generalized Floquet theory developed by T.A. Burton and J.S. Muldowney by providing a necessary and sufficient condition for h–dichotomy.

It should be noted that all the results have been obtained by using a characterization of the h–dichotomy by a group theory approach recently developed by J.F. Peña and S. Rivera–Villagrán.

Gonzalo Robledo Veloso, Department of Mathematics – University of Chile
e-mail : grobledo@uchile.cl